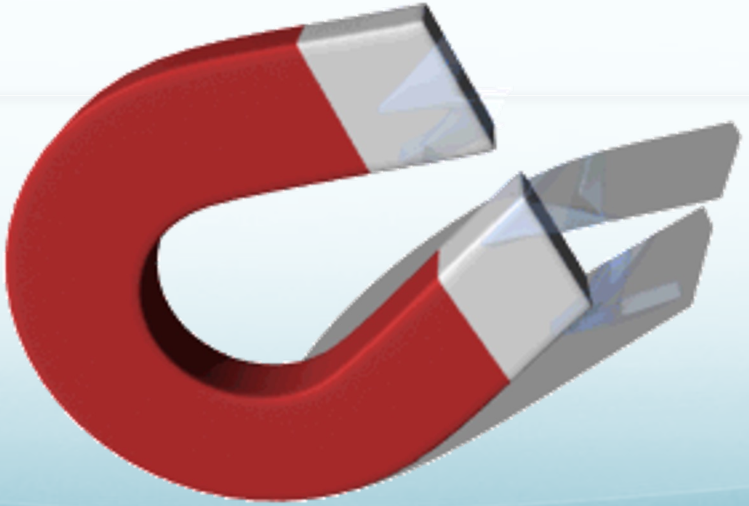
General Physics 2
SCPH211

Chapter 29
Magnetic Fields due to Currents



Outlines

- Magnetic Field due to Currents
- Force Between Two Parallel Currents
- Ampere’s Law
- Solenoids
- Current-Carrying Coil as a Magnetic Dipole

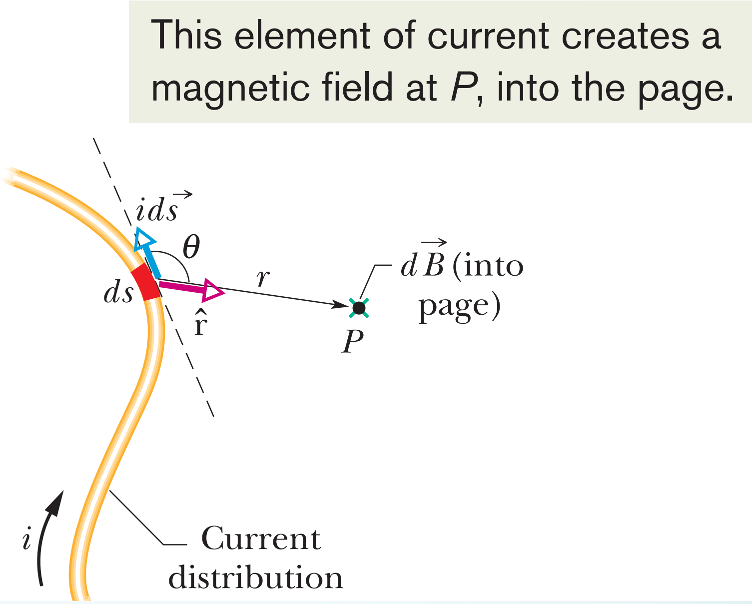
Calculating the Magnetic Field due to a Current

To find the the magnetic field produced at point *P* due to the current *i* passing through a wire of length *L*:

- The wire is divided into differential elements *ds*
- The distance between *P* and *ds* is r
- ***dB*** produced by *ds* is given by

$$dB = \frac{\mu_0}{4\pi} \frac{i\, ds \sin\theta}{r^2} = \frac{\mu_0}{4\pi} \frac{i\, d\vec{s} \times \hat{\mathrm{r}}}{r^2}$$

(Biot – Savart law).

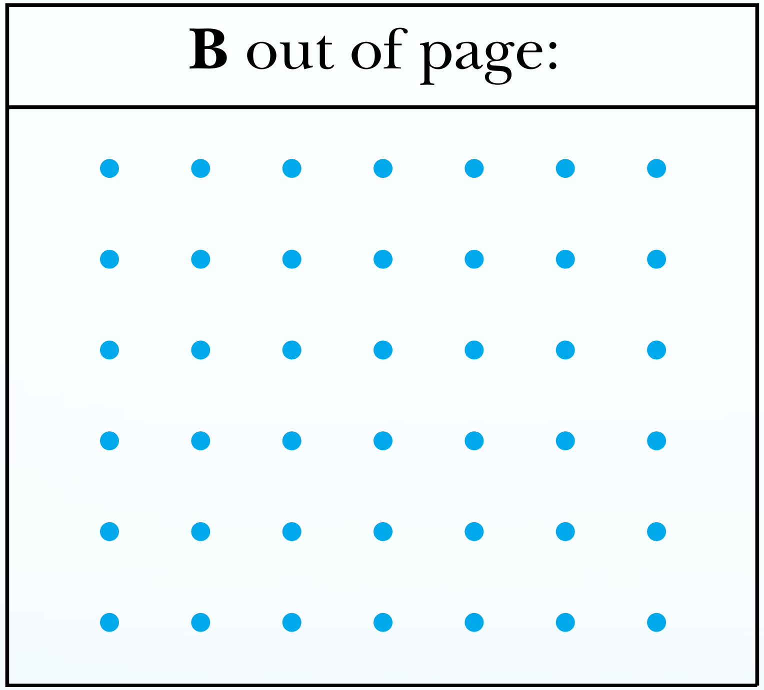


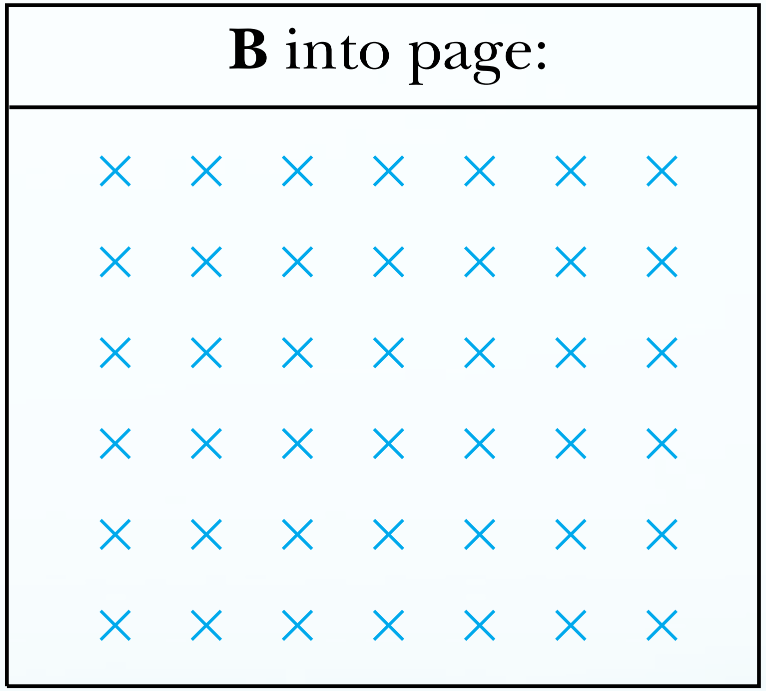
where θ : the angle between the directions of *ds* and $\hat{r}$

$\hat{r}$: the unit vector that points from *ds* toward *P*.

μ_0 : the permeability constant

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A} \approx 1.26 \times 10^{-6}\ \mathrm{T \cdot m/A}.$$



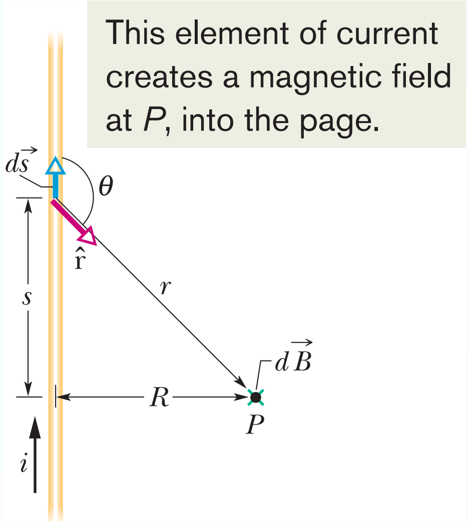


The representation of the magnetic field **B** when it is directed into the page or out of it.

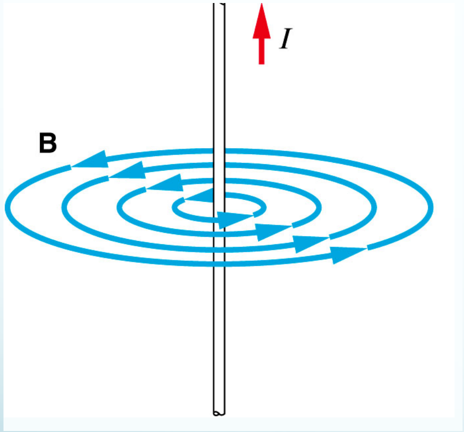
Magnetic filed due to current in a long straight wire

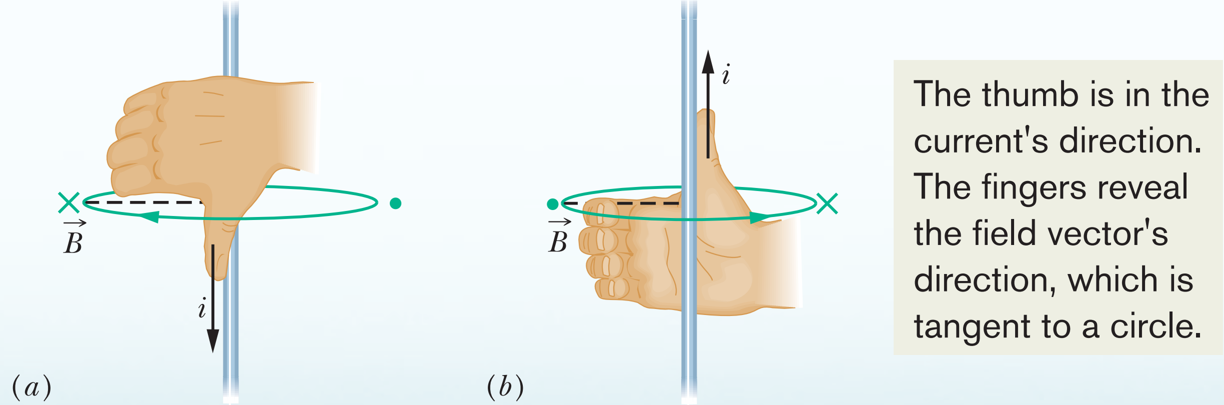
- The magnetic field at a perpendicular distance R from a long (infinite) straight wire carrying a current i is given by

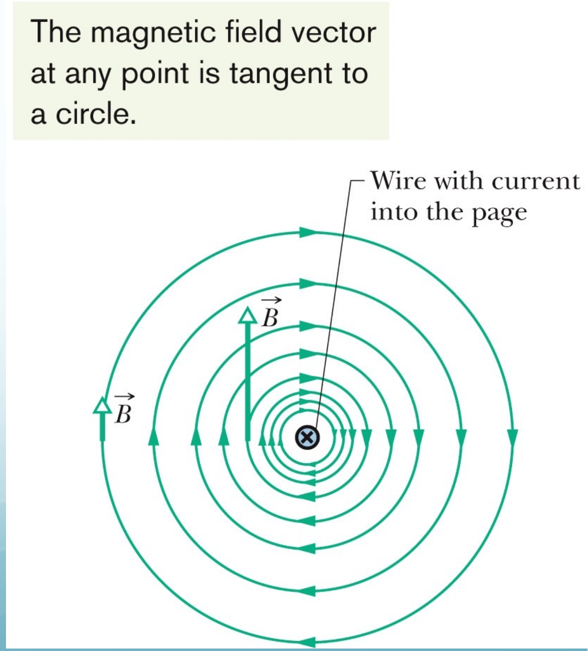
$$B = \frac{\mu_0 i}{2\pi R} \qquad \text{(long straight wire).}$$



- The direction of B is determined by the right-hand-rule:





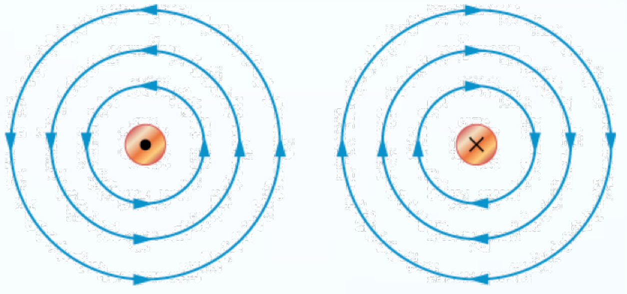


Right-hand rule: Grasp the element in your right hand with your extended thumb pointing in the direction of the current. Your fingers will then naturally curl around in the direction of the magnetic field lines due to that element.

Example:

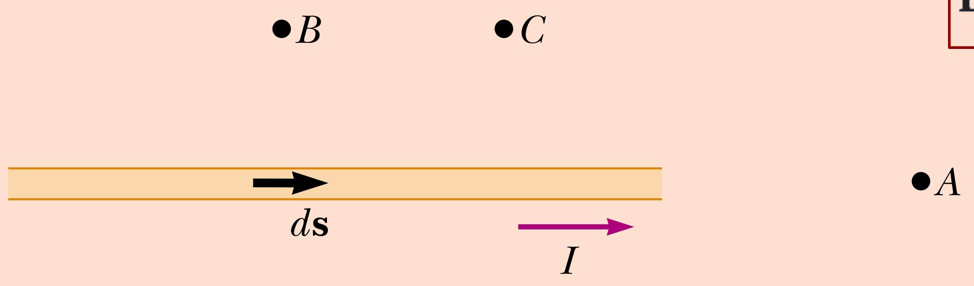
For a current carrying wire:

- If current is going into the paper plane, **B** goes in (clockwise, counter-clockwise) direction.
- If current is coming out of the paper plane, **B** goes in (clockwise, counter-clockwise) direction.



Example:

Consider the current in the length of wire shown in Figure 30.2. Rank the points *A*, *B*, and *C*, in terms of magnitude of the magnetic field due to the current in the length element shown, from greatest to least.



$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{l}' \times \hat{\boldsymbol{\imath}}}{\imath^2}.$$

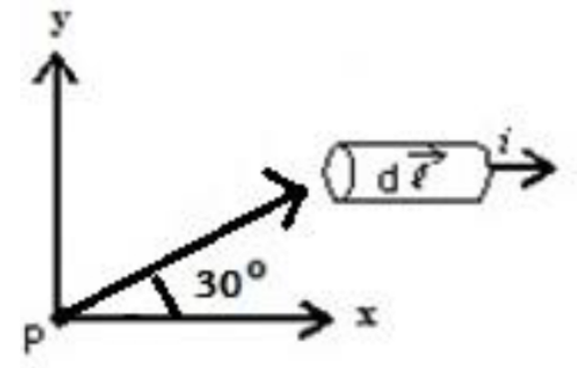
Biot-Savart law

Figure 30.2 Where is the magnetic field the greatest?

B,*C*,*A*. Point *B* is closest to the current element. Point *C* is farther away and the field is further reduced by the sin factor in the cross product $d\mathbf{s} \times \mathbf{r}$. The field at *A* is zero because $\theta = 0$.

Example 1:

In the figure, if the current element has a length of 1.0 mm, carries a current of 2.5 A, and is a distance of 4.8 cm from the point P, what is the magnitude of the magnetic field at point P?



Solution:

(A) 0 T
(B) 2.6×10^{-9} T
(C) 5.4×10^{-8} T
(D) 9.4×10^{-8} T

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\mathbf{L}' \times \hat{\mathbf{r}}}{r^2}.$$

Biot-Savart law

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

$$B = \frac{4\pi 10^{-7}}{4\pi}(2.5)\frac{0.001 \sin 30}{(4.8 \times 10^{-2})^2} = 5.4 \times 10^{-8}\mathrm{T}$$

Example 2:

The magnetic field a distance 2 cm from a long straight current-carrying wire is 2×10^{-5} T. The current in the wire is:

Solution:

(A) 1.0 A
(B) 2.0 A
(C) 4.0 A
(D) 25 A

$$B = \frac{\mu_0 i}{2\pi R} \qquad \text{(long straight wire).}$$

$$i = \frac{2\pi RB}{\mu_o} = \frac{2\pi(2\times10^{-2})(2\times10^{-5})}{4\pi\times10^{-7}} = 2\text{A}$$

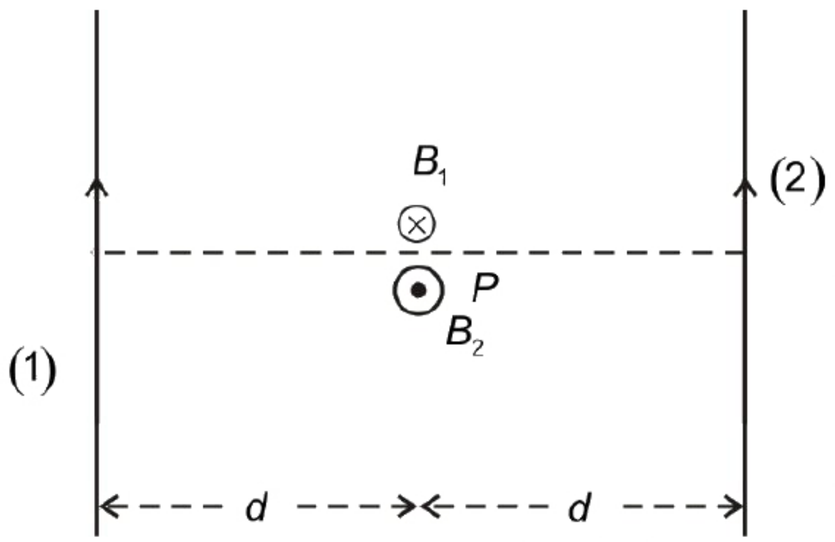
Example 3:

Two long straight wires are parallel and carry current in the same direction. The currents are 8.0 and 12 A and the wires are separated by 0.40 cm. The magnetic field at a point midway between the wires is:

Solution:

(A) 4.0×10^{-4} T
(B) 8.0×10^{-4} T
(C) 12×10^{-4} T
(D) 20×10^{-4} T

Magnetic field at point p due 1st and 2nd wire



$$B_1 = B_2 = \frac{\mu_o}{2\pi}\frac{I}{d}$$

$$B_p = B_1 + B_2 = \frac{\mu_o}{2\pi}\frac{I}{d} - \frac{\mu_o}{2\pi}\frac{I}{d}$$

$$B_1 = \frac{\mu_o i_1}{2\pi R} = \frac{4\pi\times10^{-7}\ (8)}{2\pi(0.2\times10^{-2})} = 80\times10^{-5}\text{T}$$

$$B_2 = \frac{\mu_o i_2}{2\pi R} = \frac{4\pi\times10^{-7}\ (12)}{2\pi(0.2\times10^{-2})} = 120\times10^{-5}\text{T}$$

B at the midpoint $= B_2 - B_1 = (120 - 80)\times10^{-5}$
$= 40\times10^{-5} = 4\times10^{-4}\text{T}$

Force Between Two Parallel Currents

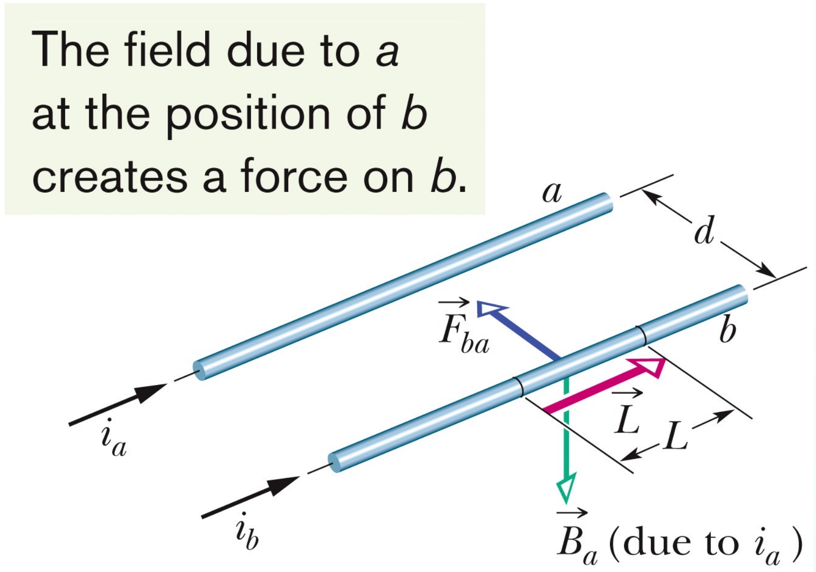
- two wires, separated by a distance d and carrying currents i_a and i_b
- The magnitude of $\mathbf{B}_a$ at every point of wire b *is:*

$$B_a = \frac{\mu_0 i_a}{2\pi d}.$$

the magnetic force $\boldsymbol{F}_{ba}$ produces on a length L of wire b due $\mathbf{B}_a$ to is:

$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a,$$

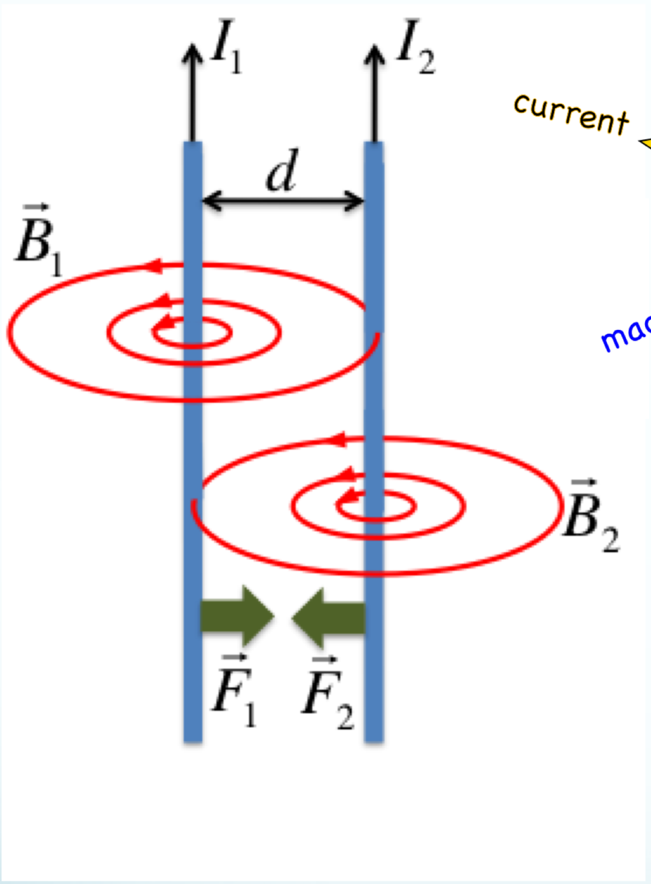
$$\rightarrow \quad F_{ba} = i_b L B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d}.$$

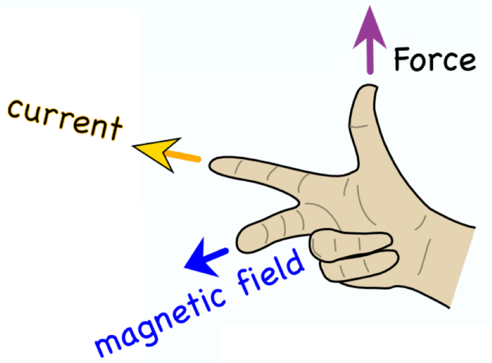


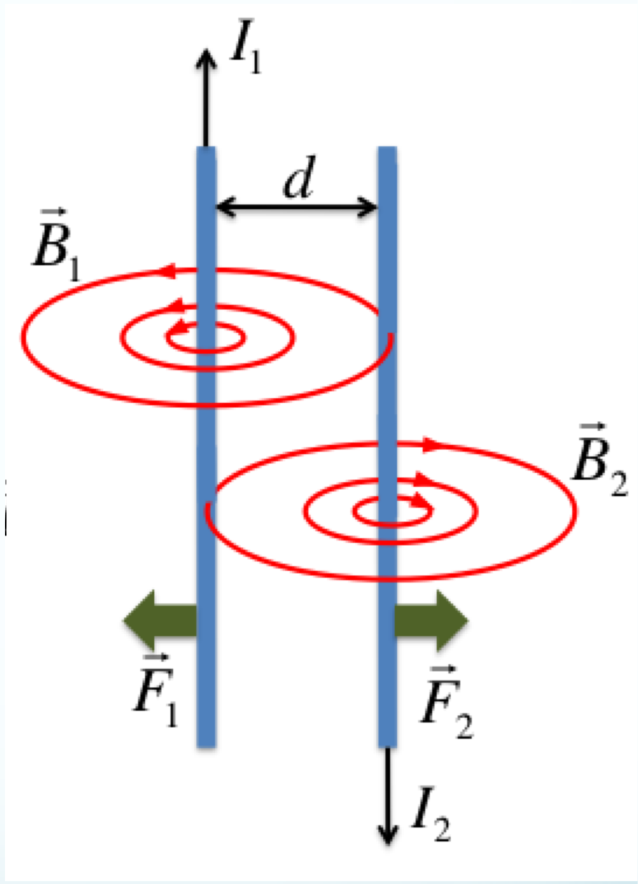
- .

To find the force on a current-carrying wire due to a second current-carrying wire, first find the field due to the second wire at the site of the first wire. Then find the force on the first wire due to that field.

Parallel currents attract each other, and antiparallel currents repel each other.







At 2nd wire, $\mathbf{B_1}$ from 1st wire points *into the page, I_2 upward,* $\mathbf{F}_2$ *to the left.*
At 1st wire, $\mathbf{B_2}$ from 2nd wire points *out the page, I_1 upward,* $\mathbf{F}_1$ *to the right.*

At 2nd wire, $\mathbf{B_1}$ from 1st wire points *into the page, I_2 downward,* $\mathbf{F}_2$ *to the right.*
At 1st wire, $\mathbf{B_2}$ from 2nd wire points *into the page, I_1 upward,* $\mathbf{F}_1$ *to the left.*

Example 4:

Two parallel wires carrying equal currents of 10 A attract each other with a force of 1 mN. If both currents are doubled, the force of attraction will be:

Solution:

(A) 2 mN
(B) 1 mN
(C) 4 mN
(D) 6 mN

$$\frac{\mu_o L}{2\pi d} i_1 i_2 = 1\text{mN}$$

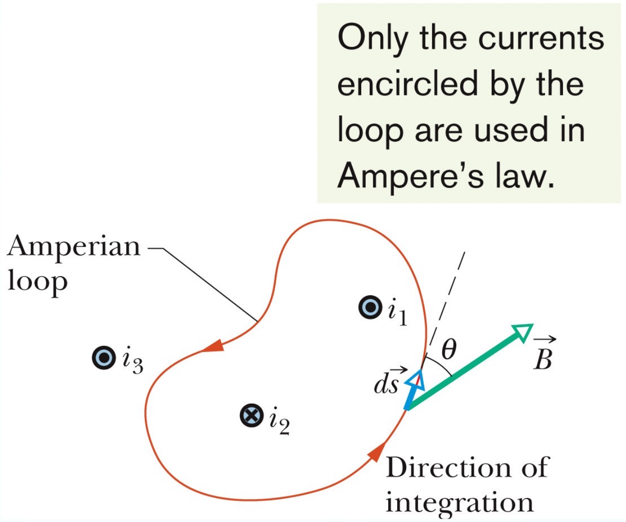
$$\frac{\mu_o L}{2\pi d} 2i_1 2i_2 = ?$$

→ $F = 4\text{mN}$

Ampere's Law

- **Ampere's law** is used to find the magnetic field due to a current with less effort than using Biot – Savart law,

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law).}$$



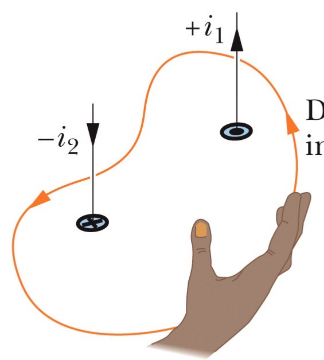
- **Ampere's Law states that:** along any arbitrary path encircling a total current I_{enc}, the integral of the scalar product of the magnetic field **B** with the element of length *d***s** of the path, is equal to the permeability μ_o times the total current i_{enc} enclosed by the path.

- Applying Ampere's law:
  - Indicate which current in the problem is under question.
  - Draw a closed loop called (Amperian loop) enclosing the current under question.
  - Apply the integral above (Amper's law).
  - The integral of the scalar product of the magnetic field **B** with the element of length *d***s** of the path, is equal to the permeability μ_o times the total current i_{enc} enclosed by the path.

- **How to determining the +ve & -ve current when applying Ampere's law:**

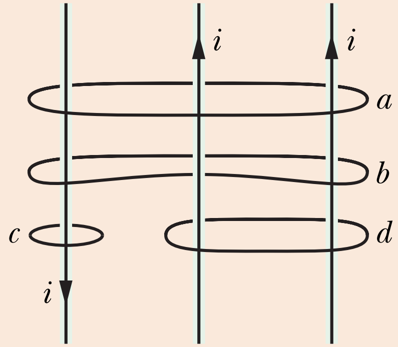
Curl your right hand around the Amperian loop, with the fingers pointing in the direction of integration. A current through the loop in the general direction of your outstretched thumb is assigned a plus sign, and a current generally in the opposite direction is assigned a minus sign.

This is how to assign a sign to a current used in Ampere's law.



CHECKPOINT 2

The figure here shows three equal currents i (two parallel and one antiparallel) and four Amperian loops. Rank the loops according to the magnitude of $\oint \vec{B} \cdot d\vec{L}$ along each, greatest first.



$$\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 I_{enc}.$$

$d=2i$

Then a & c tie $=i$

Then $b=0$

Example 5:

A long straight wire carrying a 3.0 A current enters a room through a window 1.5 m high and 1.0 m wide. The path integral $\oint \boldsymbol{B} \cdot d\boldsymbol{s}$ around the window frame has the value:

Solution:

(A) 0.20 T.m
(B) 2.5×10^{-7} T.m
(C) 3.0×10^{-7} T.m
(D) 3.8×10^{-6} T.m

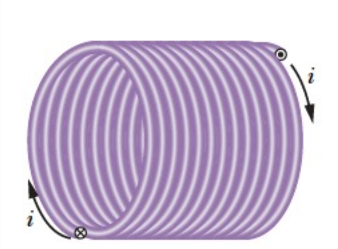
$$\oint \mathbf{B} \cdot d\mathbf{L} = \mu_0 I_{\text{enc}}.$$

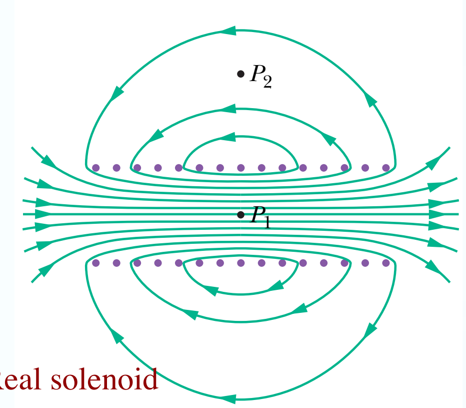
$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$

$$\oint \boldsymbol{B} \cdot d\boldsymbol{L} = \mu_o \times I = 4(3.14) \times 10^{-7}(3) = 3.8 \times 10^{-6} T$$

Solenoids

Magnetic Field of a Solenoid





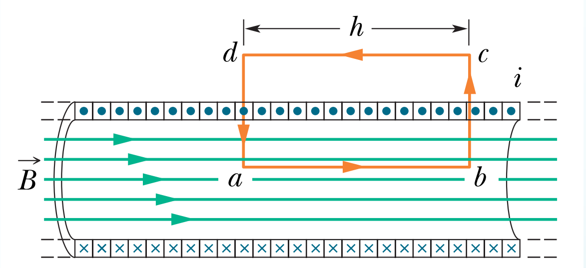
Real solenoid

Solenoid: long coil of wire with many turns

- We have infinitely long solenoid of *n* turns (loops) per unit length ($n = N/h$) (N: total number of turns around a cylinder of radius R)
- Using the rectangular Amperian loop *abcda*

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_o i_{\text{enc}}$$

- Because the rectangular Amperian loop has N turns → $i_{\text{enc}} = N\,i = i\,nh$ ($N = nh$)



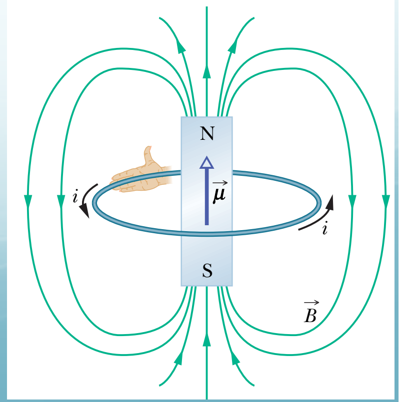
$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_o i_{\text{enc}} = \mu_o nih \rightarrow \oint \mathbf{B} \cdot d\mathbf{s} = \int_{AB} \boldsymbol{B} \cdot d\boldsymbol{s} + \int_{BC} \boldsymbol{B} \cdot d\boldsymbol{s} + \int_{CD} \boldsymbol{B} \cdot d\boldsymbol{s} + \int_{DA} \boldsymbol{B} \cdot d\boldsymbol{s}$$

↓ Bh? ↓ zero? ↓ zero? ↓ zero?

$$\rightarrow \oint \mathbf{B} \cdot d\mathbf{s} = Bh = \mu_o nIh$$

$$B = \mu_0 in \qquad \text{(ideal solenoid).}$$

- The B field is strong and uniform at interior points P_1 but weak or zero at external points P_2
- The magnetic field produced by a current-carrying coil, is called a **magnetic dipole**



Example 6:

Solenoid 2 has twice the radius and six times the number of turns per unit length as solenoid 1. The ratio of the magnetic field in the interior of 2 to that in the interior of 1 is:

Solution:

(A) 1
(B) 2
(C) 4
(D) 6

$$B = \mu_0 in \qquad \text{(ideal solenoid).}$$

$$\frac{B_2}{B_!} = \frac{\mu_o i6n}{\mu_o in} = 6$$

Example 7:

A solenoid is 3.0 cm long and has a radius of 0.50 cm. It is wrapped with 500 turns of wire carrying a current of 2.0 A. The magnetic field at the center of the solenoid is:

Solution:

(A) 9.9×10^{-8} T
(B) 1.3×10^{-3} T
(C) 4.2×10^{-2} T
(D) 16 T

$B = \mu_0 in$ (ideal solenoid).

$$n = \frac{N}{h} = \frac{500}{3\times10^{-2}} = 16{,}666.7$$

$$B = \mu_o in = 4(3.14)\times10^{-7}(2)(16{,}666.7) = 0.042\,T$$