General Physics 2
SCPH 211

Chapter 25
CAPACITANCE

▲ *All of these devices are capacitors, which store electric charge and energy. A capacitor is one type of circuit element that we can combine with others to make electric circuits.*
(Paul Silverman/Fundamental Photographs)

Outline:

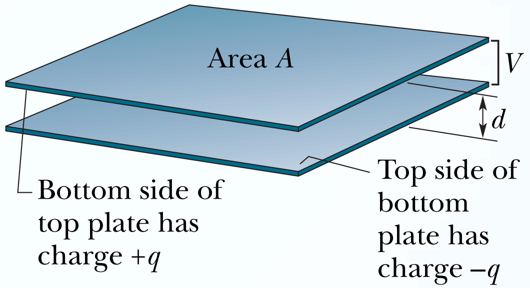
- Introduction
- Capacitor & Capacitance
- Charging a Capacitor
- Calculating the Capacitance
- Capacitors in Parallel and in Series
- Energy Stored in an Electric Field

Introduction

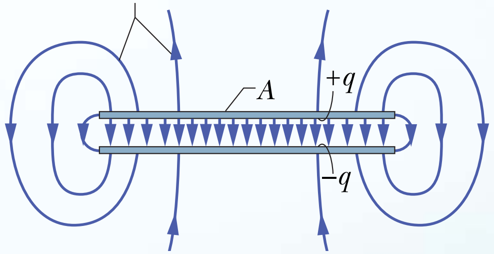
- ***Capacitor*:** a device in which electrical energy can be stored. e.g., the batteries in a camera store energy in the photoflash unit by charging a capacitor
- The physics of capacitors can be generalized to other devices and to any situation involving electric fields.
  - e.g., Earth's atmospheric electric field is modeled as being produced by a huge spherical capacitor that partially discharges via lightning
- Our discussion of capacitors
  - To *determine how much charge can be stored*
  - This "how much" is called capacitance

Introduction

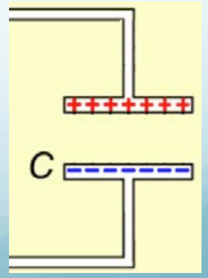
- ***Capacitor***: consists of two isolated conductors, called *plates*, of any shape



- A *parallel-plate capacitor,* consisting of two parallel conducting plates of area A separated by a distance d
- The symbol of a capacitor in electric circuit 



- When a capacitor is *charged*, its plates have charges of equal magnitudes but opposite signs $+q$ & $-q$
  - We refer to the *charge of a capacitor* as q
  - The net charge on the capacitor is zero
- Because the plates are conductors
  → they are equipotential surfaces
  → all points on a plate are at the same V
  - There is a potential difference between the two plates represented V rather than ΔV

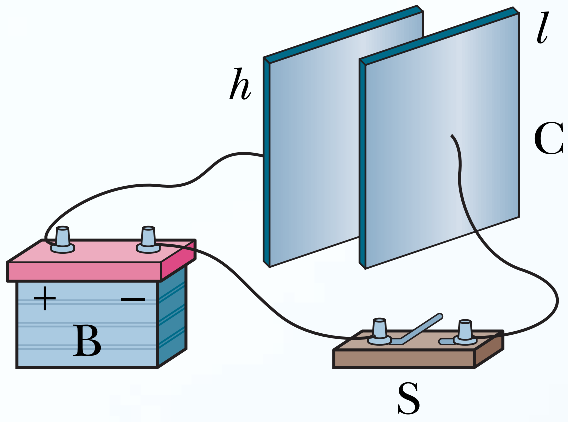


Capacitor

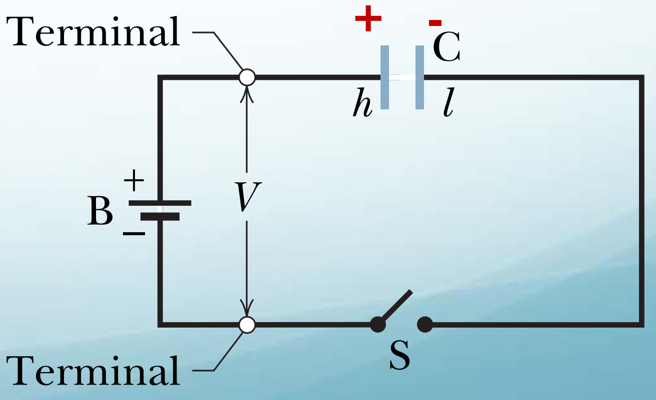
- q & V for a capacitor are proportional to each other $\boxed{q = CV.}$

Where C is called the ***capacitance*** of the capacitor

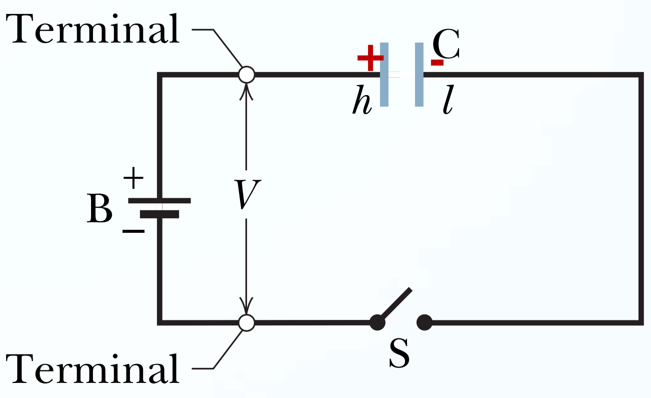
- C depends on the geometry of the plates and *not* on q or V
- The greater the capacitance, the more charge is required to produce a certain potential difference between them
- The SI unit of C is **farad** (**F**): **1 farad = 1coulomb/volt** (pF $=10^{-12}$ F, μF $=10^{-6}$F)



- **Charging a Capacitor:** by connecting the plates with a battery, a potential difference between capacitor *terminals is* maintained
  - The terminal of higher potential is + (*positive* terminal)
    The terminal of lower potential is – (*negative* terminal)
  - When switch S is *closed*
    → charges (electrons) flow through the wire
  - E from battery drives electrons from plate h to +ve terminal of the battery
    → plate h loses electrons → becomes positively charged
  - E drives electrons from –ve terminal of the battery to plate l
    → plate l gains electrons → becomes negatively charged
  - <u>(*as much* as plate h, losing electrons, plate l gaining)</u>



- When plates are uncharged → V between them = zero
- During charging, potential difference V increases until becomes equals to potential difference V of the battery
- → plate h & +ve terminal of the battery have same potential
  → no electric field in the wire between them
- Similarly, plate l & -ve terminal reach the same potential
  → no electric field in the wire between them
- When E = zero between battery and plates
  → no further drive of electrons
  → capacitor is *fully charged*



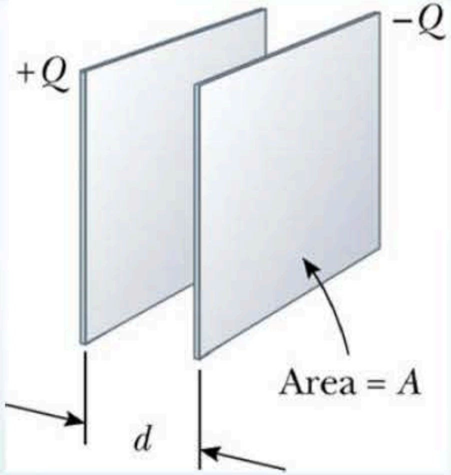
CHECKPOINT 1

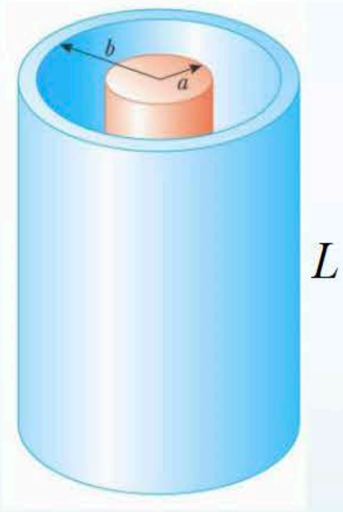
Does the capacitance C of a capacitor increase, decrease, or remain the same (a) when the charge q on it is doubled and (b) when the potential difference V across it is tripled?

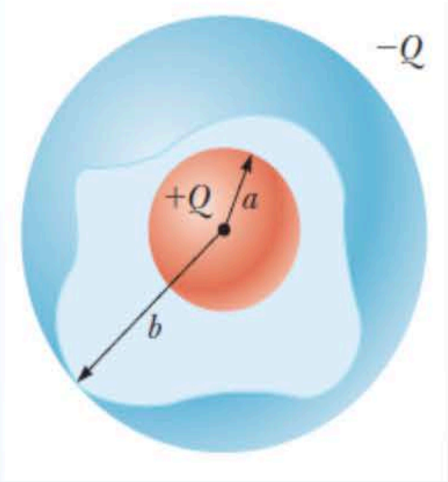
(a) same;
(b) same

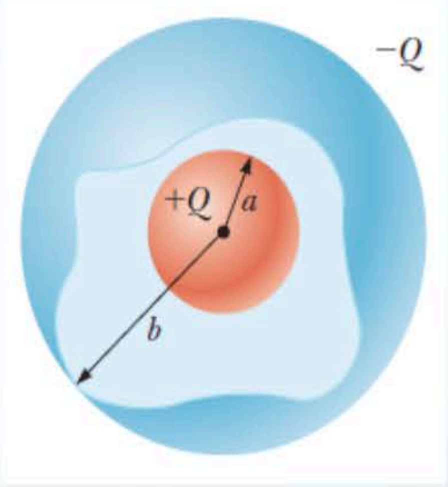
Capacitance

- Parallel plate
- Cylindrical
- Spherical
- Isolated sphere



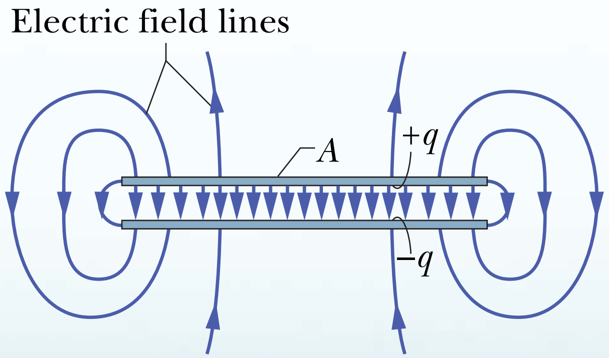






Calculating the Capacitance

1. Assume a charge = q on the plates
2. Calculate $\boldsymbol{E}$ between the plates using Gauss' law → $\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}}$
3. Calculate V between the plates → $\boxed{V = -\int_i^f \vec{E} \cdot d\vec{s}}$
4. Calculate C → $\boxed{q = CV.}$



Calculating the Electric Field:

- $\boldsymbol{E}$ and $\boldsymbol{dA}$ is parallel→ $\boxed{q = \varepsilon_0 EA}$
  → $E = q/\varepsilon_0 A$
- ***Calculating the Potential Difference:***
- We choose a path follows electric field line from –ve to +ve plate
  - For this path, the vectors $\boldsymbol{E}$ & $\boldsymbol{ds}$ have opposite directions → $V = \int_{-}^{+} E\, ds$

A Parallel-Plate Capacitor

A: plate area, d distance between plates

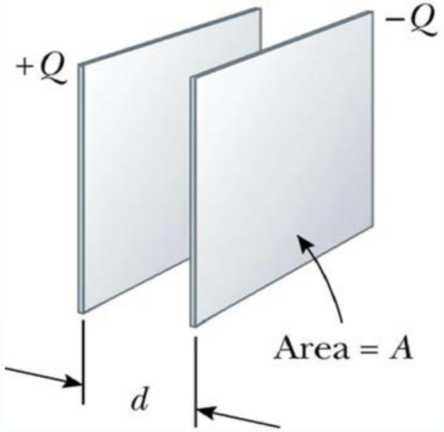
$$\boxed{q = \varepsilon_0 E A}$$

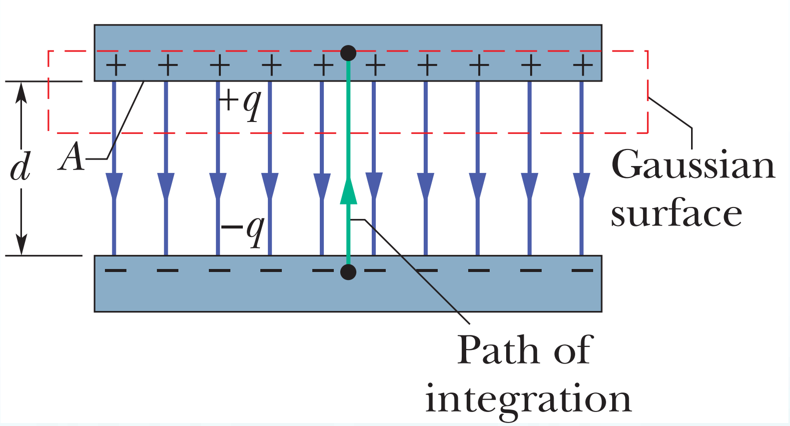
$$\boxed{V = \int_{-}^{+} E\, ds}$$

$$= E\int_{-}^{+} ds = E\, d$$

$$C = \frac{q}{V} = \frac{\varepsilon_o E A}{E d} = \frac{\varepsilon_o A}{d}$$

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{(parallel-plate capacitor).}$$





Problems:

1. **A parallel-plate capacitor with plate's area of $25cm^2$ and separation of 17.7mm is charged by applying a voltage of 12V across its ends. Calculate the total charge of the capacitor.**

$$C = \frac{A\varepsilon_0}{d} = \frac{25\times10^{-4}\times8.85\times10^{-12}}{17.7\times10^{-3}} = 1.25pF$$

$$q = CV = 1.25\times10^{-12}\times12 = 15\mathrm{pC}$$

Problems:

2. **A parallel plate capacitor has a capacitance of 8μF. Calculate the capacitance if the**

   i. **Plate separation is doubled**

$C = \frac{A\varepsilon_0}{d} = 8\mu\text{F}$

When $d' = 2d$ → $C' = \frac{A\varepsilon_0}{2d} = \frac{8}{2} = 4\mu\text{F}$

   ii. **Plate area is doubled**

   When $A' = 2A$ → $C' = \frac{2A\varepsilon_0}{d} = 2(8) = 16\mu\text{F}$

CHECKPOINT 2

For capacitors charged by the same battery, does the charge stored by the capacitor increase, decrease, or remain the same in each of the following situations? (a) The plate separation of a parallel-plate capacitor is increased. (b) The area of the plate of a parallel-plate capacitor is increased.

$$C = \frac{\varepsilon_0 A}{d} \qquad \text{(parallel-plate capacitor).}$$

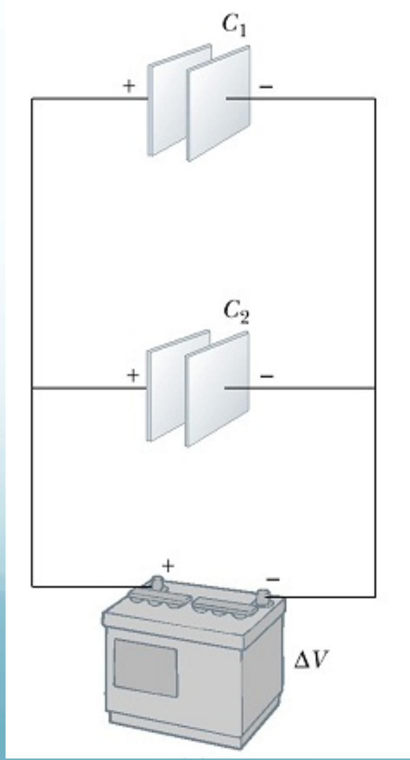
(a) decreases;
(b) increases;

Capacitors in Parallel and in Series

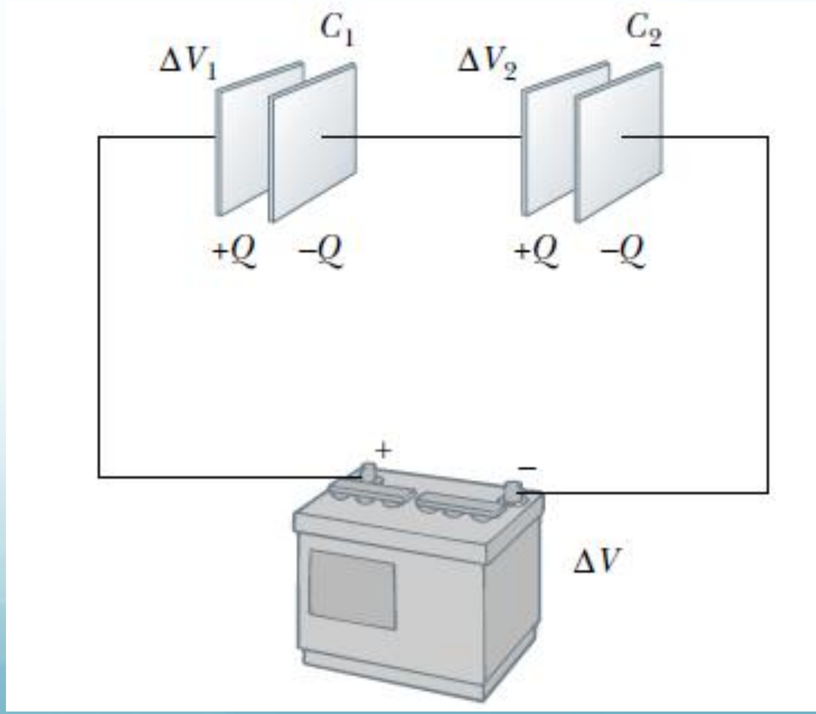
- To simplify circuit, we replace a combination of capacitors in a circuit, with an ***equivalent capacitor***
- ***Equivalent capacitor:*** a single capacitor that has the same capacitance as the actual combination of capacitors

Capacitors Combinations

Parallel



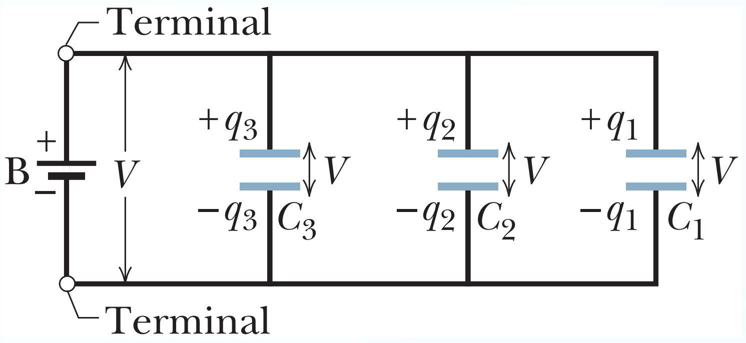
Series



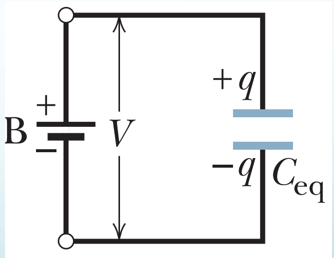
Capacitors in Parallel

- "in parallel" means that the capacitors are directly wired together at one plate and directly wired together at the other plate
- Each capacitor has same V, which produces a charge on each capacitor

When a potential difference V is applied across several capacitors connected in parallel, that potential difference V is applied across each capacitor. The total charge q stored on the capacitors is the sum of the charges stored on all the capacitors.



Capacitors connected in parallel can be replaced with an equivalent capacitor that has the same *total* charge q and the same potential difference V as the actual capacitors.



$$q_1 = C_1V, \quad q_2 = C_2V, \quad \text{and} \quad q_3 = C_3V.$$

$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3,$$

$$\boxed{C_{eq} = \sum_{j=1}^{n} C_j \qquad (n \text{ capacitors in parallel}).}$$

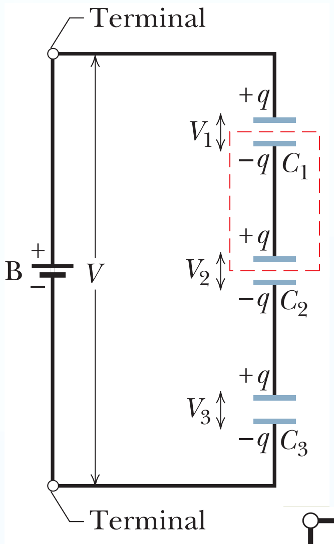
$C_{eq} > C_1, C_2, C_3, \ldots..$

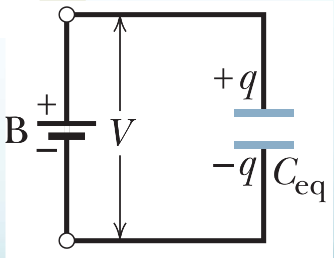
Capacitors in Series

- "in series" means that the capacitors are wired serially, one after the other
- The potential differences that then exist across the capacitors in the series produce identical charges q on them

When a potential difference V is applied across several capacitors connected in series, the capacitors have identical charge q. The sum of the potential differences across all the capacitors is equal to the applied potential difference V.

Capacitors that are connected in series can be replaced with an equivalent capacitor that has the same charge q and the same *total* potential difference V as the actual series capacitors.





$$V_1 = \frac{q}{C_1}, \quad V_2 = \frac{q}{C_2}, \quad \text{and} \quad V_3 = \frac{q}{C_3}.$$

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

$$C_{eq} = \frac{q}{V} = \frac{1}{1/C_1 + 1/C_2 + 1/C_3},$$

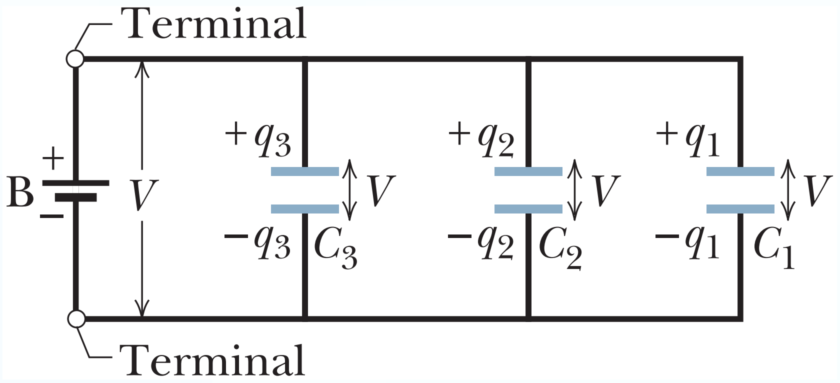
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \qquad (n \text{ capacitors in series}).$$

$C_{eq} < C_1, C_2, C_3, \ldots\ldots$

Capacitors Combinations

Parallel



V is constant

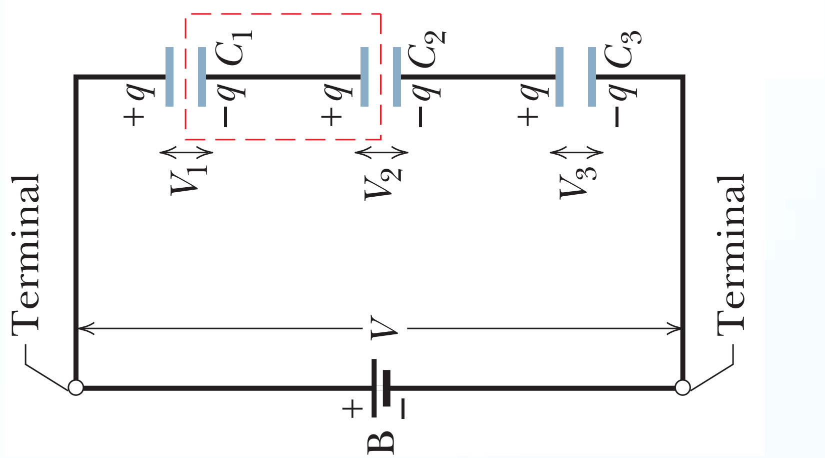
$$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V.$$

$$C_{eq} = C_1 + C_2 + C_3,$$

$$\boxed{C_{eq} = \sum_{j=1}^{n} C_j \qquad (n \text{ capacitors in parallel}).}$$

$C_{eq} > C_1, C_2, C_3, \ldots\ldots$

Series



q or I is constant

$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right).$$

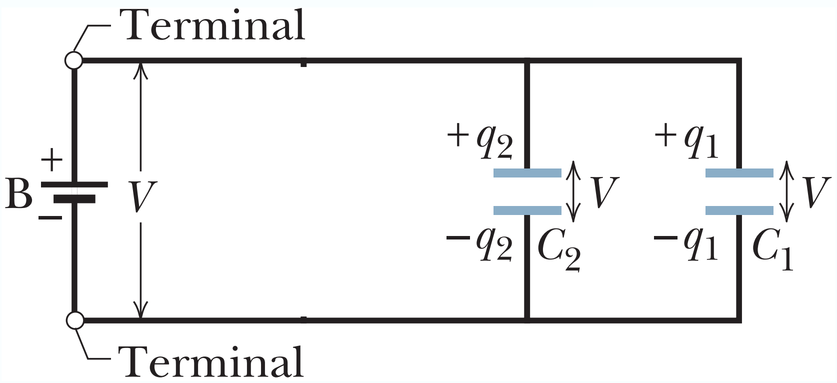
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

$$\boxed{\frac{1}{C_{eq}} = \sum_{j=1}^{n} \frac{1}{C_j} \qquad (n \text{ capacitors in series}).}$$

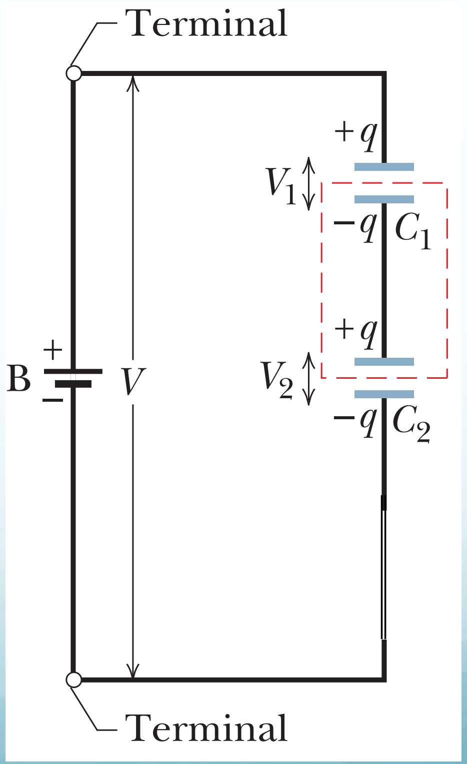
$C_{eq} < C_1, C_2, C_3, \ldots\ldots$

CHECKPOINT 3

A battery of potential V stores charge q on a combination of two identical capacitors. What are the potential difference across and the charge on either capacitor if the capacitors are (a) in parallel and (b) in series?



(a) V, q/2;
(b) V/2; q



Sample Problem

Capacitors in parallel and in series

(a) Find the equivalent capacitance for the combination of capacitances shown in Fig. 25-10*a*, across which potential difference V is applied. Assume

$$C_1 = 12.0\ \mu\text{F}, \quad C_2 = 5.30\ \mu\text{F}, \quad \text{and} \quad C_3 = 4.50\ \mu\text{F}.$$

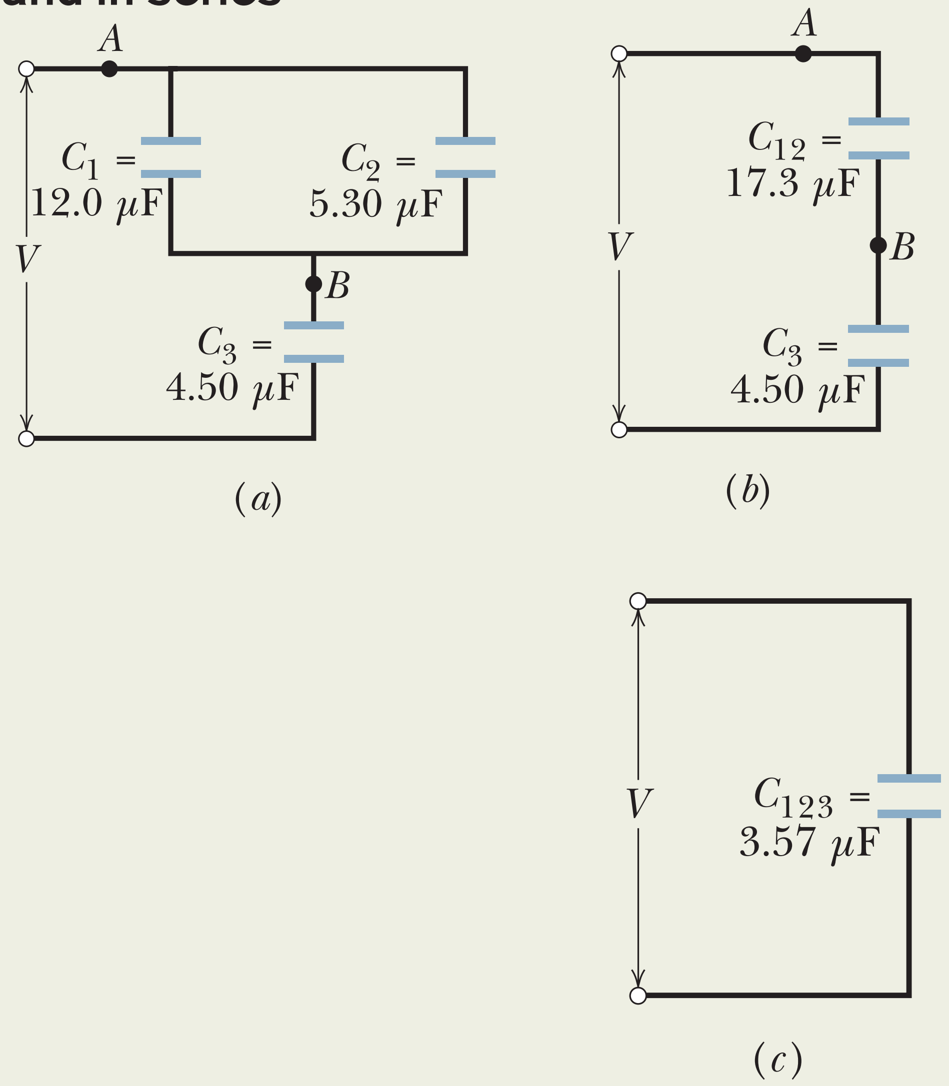
$$C_{12} = C_1 + C_2 = 12.0\ \mu\text{F} + 5.30\ \mu\text{F} = 17.3\ \mu\text{F}.$$

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3}$$

$$= \frac{1}{17.3\ \mu\text{F}} + \frac{1}{4.50\ \mu\text{F}} = 0.280\ \mu\text{F}^{-1},$$

from which

$$C_{123} = \frac{1}{0.280\ \mu\text{F}^{-1}} = 3.57\ \mu\text{F}. \qquad \text{(Answer)}$$



(b) The potential difference applied to the input terminals in Fig. 25-10*a* is $V = 12.5$ V. What is the charge on C_1?

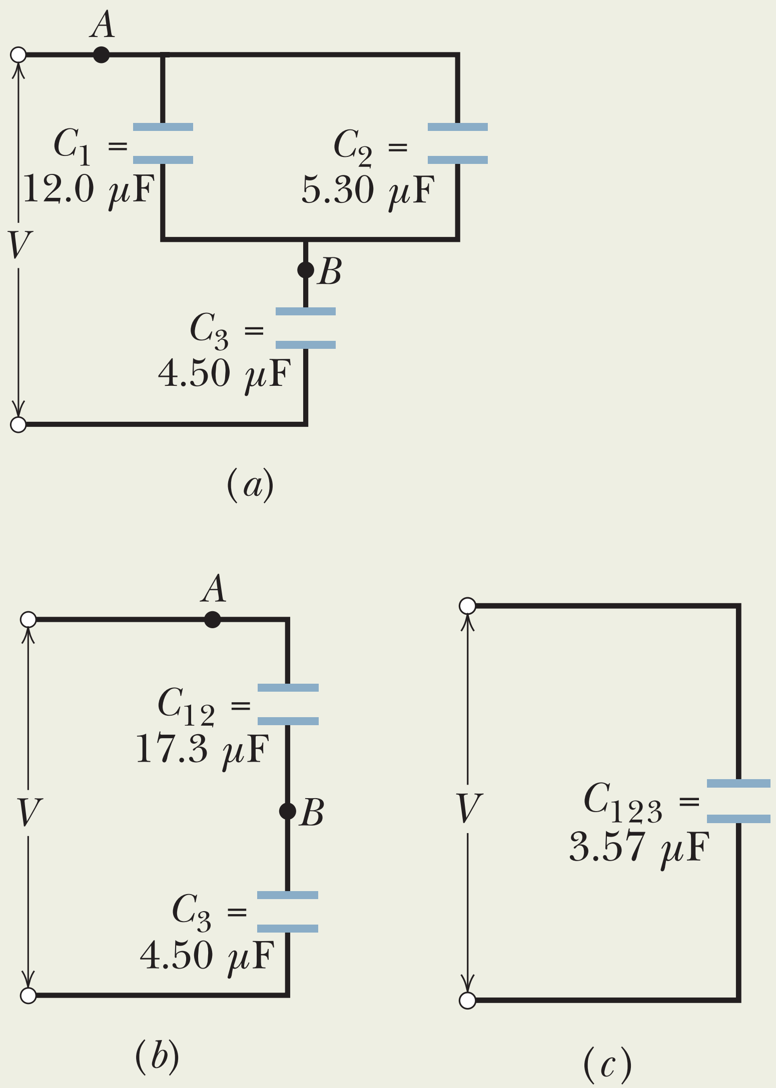
$$q_{123} = C_{123}V = (3.57\ \mu\text{F})(12.5\ \text{V}) = 44.6\ \mu\text{C}.$$

The series capacitors 12 and 3 in Fig. 25-10*b* each have the same charge as their equivalent capacitor 123 (Fig. 25-10*f*). Thus, capacitor 12 has charge $q_{12} = q_{123} = 44.6\ \mu\text{C}$.

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\ \mu\text{C}}{17.3\ \mu\text{F}} = 2.58\ \text{V}.$$

The parallel capacitors 1 and 2 each have the same potential difference as their equivalent capacitor 12 (Fig. 25-10*h*). Thus, capacitor 1 has potential difference $V_1 = V_{12} = 2.58$ V,

$$q_1 = C_1V_1 = (12.0\ \mu\text{F})(2.58\ \text{V})$$
$$= 31.0\ \mu\text{C}. \qquad \text{(Answer)}$$

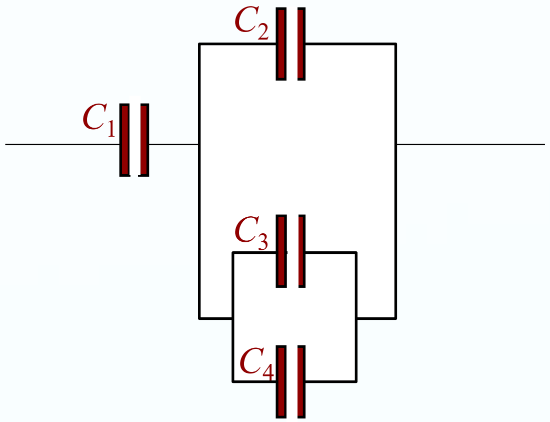


Problems:

6. In this figure, $C_1 = 6\mu F$ and $C_2 = C_3 = C_4 = 2\mu F$. Calculate the equivalent capacitance.

C_2, C_3 & C_4 are in parallel → $C_{234} = 2 + 2 + 2 = 6\ \mu F$

C_1 & C_{234} are in series → $C_{eq} = \frac{C_1 \times C_{234}}{C_1 + C_{234}} = \frac{6\times 6}{6+6} = 3\mu F$



7. Calculate the equivalent capacitance this figure,

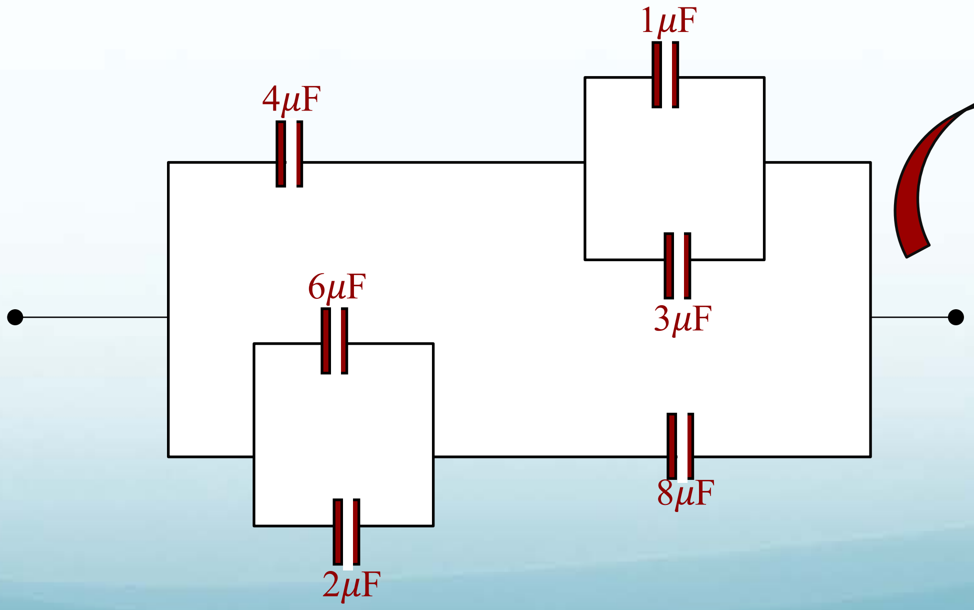
$1\mu F + 3\mu F = 4\mu F$

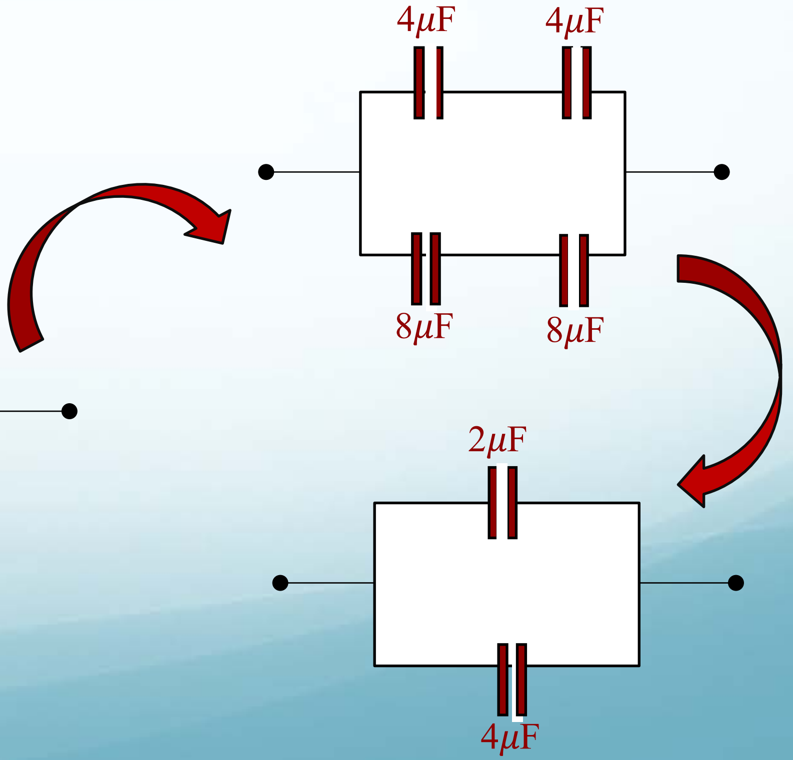
$6\mu F + 2\mu F = 8\mu F$

$\frac{4\times 4}{4+4} = 2\mu F$

$\frac{8\times 8}{8+8} = 4\mu F$

$2\mu F + 4\mu F = 6\mu F$





Problems:

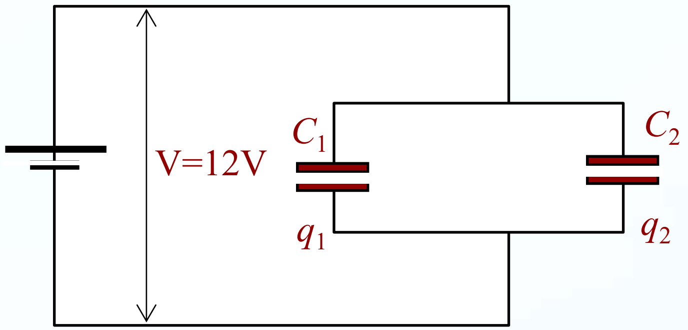
8. **In this figure, $C_1 = 6\mu$F, $C_2 = 2\mu$F and V = 12V. Calculate:**

   i. **Their equivalent capacitance.**

   $$C_{eq} = C_1 + C_2 = 6 + 2 = 8\ \mu\text{F}$$

   ii. **The charge on capacitor C_2.**

   $$q_2, = C_2\ \text{V} = 2 \times 10^{-6} \times 12 = 24 \times 10^{-6}\ \text{C}$$



Energy Stored in an Electric Field

- To charge a capacitor, work must be done by an external agent (battery)
- We visualize the work required to charge a capacitor as being stored in the form of electric potential energy U in the electric field between the capacitor plates
- To calculate the energy stored in capacitor:
  - Suppose that, at a given instant, a charge q' has been transferred from one plate of a capacitor to the other
  - The potential difference V' between the plates at that instant will be q'/C
  - If an extra increment of charge dq' is then transferred, the increment of work required will be

$$dW = V'\,dq' = \frac{q'}{C}\,dq'.$$

$$W = \int dW = \frac{1}{C}\int_0^q q'\,dq' = \frac{q^2}{2C}.$$

- This work is stored as potential energy U given by:

$$\boxed{U = \frac{q^2}{2C} \quad \text{(potential energy).}}$$ or $$\boxed{U = \tfrac{1}{2}CV^2 \quad \text{(potential energy).}}$$ or $$\boxed{U = \tfrac{1}{2}qV \quad \text{(potential energy).}}$$

The potential energy of a charged capacitor may be viewed as being stored in the electric field between its plates.

Problems:

9. **An isolated sphere is completely charged to 60pC when a voltage of 12V is applied. Calculate**

   i. **The radius of the sphere**

$$q = CV \Longrightarrow C = \frac{q}{V} = \frac{60\times10^{-12}}{12} = 5\times10^{-12}\mathrm{F}$$

$$C = 4\pi\varepsilon_0 R \Longrightarrow R = \frac{C}{4\pi\varepsilon_0} = \frac{5\times10^{-12}}{4(3.14)(8.85\times10^{-12})} = 4.5cm$$

   ii. **The energy stored within the sphere**

$$U = \frac{1}{2}qV = \frac{1}{2}\times12\times60\times10^{-12} = 360\times10^{-12}\mathrm{J}$$

1 Figure 25-18 shows plots of charge versus potential difference for three parallel-plate capacitors that have the plate areas and separations given in the table. Which plot goes with which capacitor?

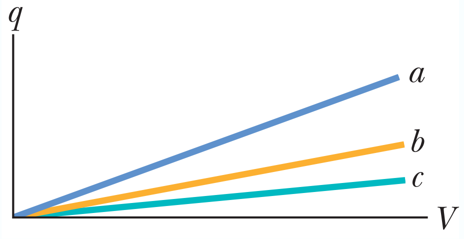


Fig. 25-18 Question 1.

Capacitor	Area	Separation
1	A	d
2	$2A$	d
3	A	$2d$

a,2;b,1;c,3

7 For each circuit in Fig. 25-21, are the capacitors connected in series, in parallel, or in neither mode?

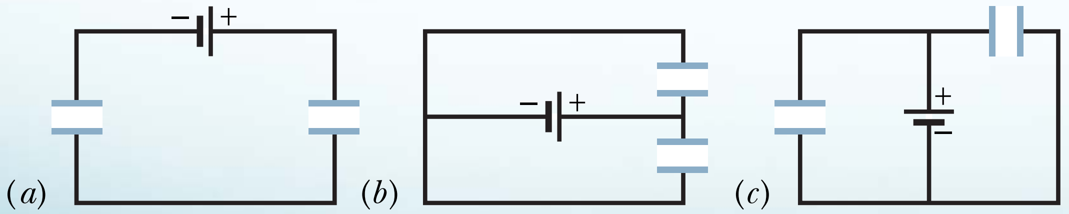


Fig. 25-21 Question 7.

11 You are to connect capacitances C_1 and C_2, with $C_1 > C_2$, to a battery, first individually, then in series, and then in parallel. Rank those arrangements according to the amount of charge stored, greatest first.

a, series; *b*, parallel; *c*, parallel

parallel, C_1 alone, C_2 alone, series

•2 The capacitor in Fig. 25-25 has a capacitance of 25 μF and is initially uncharged. The battery provides a potential difference of 120 V. After switch S is closed, how much charge will pass through it?

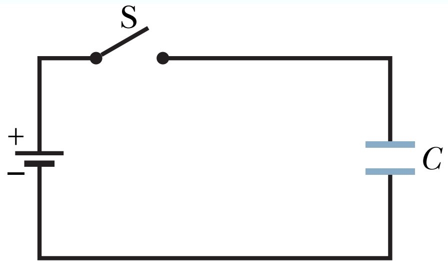


Fig. 25-25 Problem 2.

•3 SSM A parallel-plate capacitor has circular plates of 8.20 cm radius and 1.30 mm separation. (a) Calculate the capacitance. (b) Find the charge for a potential difference of 120 V.

•9 Each of the uncharged capacitors in Fig. 25-27 has a capacitance of 25.0 μF. A potential difference of $V = 4200$ V is established when the switch is closed. How many coulombs of charge then pass through meter A?

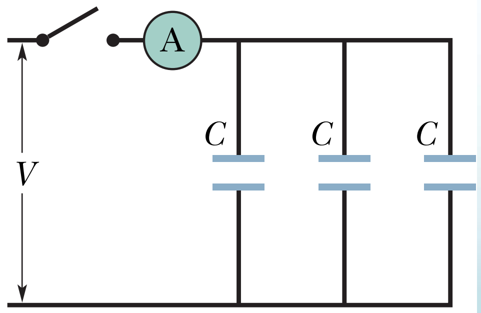


Fig. 25-27 Problem 9.

••25 In Fig. 25-40, two parallel-plate capacitors (with air between the plates) are connected to a battery. Capacitor 1 has a plate area of 1.5 cm^2 and an electric field (between its plates) of magnitude 2000 V/m. Capacitor 2 has a plate area of 0.70 cm^2 and an electric field of magnitude 1500 V/m. What is the total charge on the two capacitors?

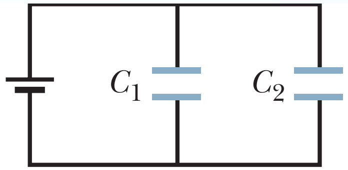


Fig. 25-40 Problem 25.

•29 What capacitance is required to store an energy of 10 kW · h at a potential difference of 1000 V?

•31 SSM A 2.0 μF capacitor and a 4.0 μF capacitor are connected in parallel across a 300 V potential difference. Calculate the total energy stored in the capacitors.