General Physics 2
SCPH 211

Chapter 24

ELECTRIC POTENTIAL

▲ Processes occurring during thunderstorms cause large differences in electric potential between a thundercloud and the ground. The result of this potential difference is an electrical discharge that we call lightning, such as this display over Tucson, Arizona. (© Keith Kent/ Photo Researchers, Inc.)

Outline:

- Electric Potential Energy
- Electric Potential
- Equipotential Surfaces
- Calculating the Potential from the Field
- Potential Due to
  - Point charge
  - Group of point charges
- Calculating the Field from the Potential
- Electric Potential Energy of a System of Point Charges
- Potential of a Charged Isolated Conductor

Electric Potential Energy

- If $\boldsymbol{F}$ acting between q_1 & q_2 → **electric potential energy** U
- ***Electric potential energy*** *U:* the energy needed to change the spatial configuration of charged particles from initial to final destination
- If the system changes from initial state i to final state f
  → work W on the particles
  → change in the potential energy $\Delta U = U_f - U_i = -W$
- The work done by the electrostatic force is ***path independent***
  → W is the same for *all* paths between points i and f
- We set the *reference potential energy* to be zero

Sample Problem

Work and potential energy in an electric field

Electrons are continually being knocked out of air molecules in the atmosphere by cosmic-ray particles coming in from space. Once released, each electron experiences an electrostatic force $\vec{F}$ due to the electric field $\vec{E}$ that is produced in the atmosphere by charged particles already on Earth. Near Earth's surface the electric field has the magnitude $E = 150$ N/C and is directed downward. What is the change ΔU in the electric potential energy of a released electron when the electrostatic force causes it to move vertically upward through a distance $d = 520$ m (Fig. 24-1)?

KEY IDEAS

(1) The change ΔU in the electric potential energy of the electron is related to the work W done on the electron by the electric field. Equation 24-1 ($\Delta U = -W$) gives the relation.

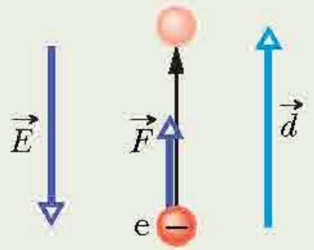

Fig. 24-1 An electron in the atmosphere is moved upward through displacement $\vec{d}$ by an electrostatic force $\vec{F}$ due to an electric field $\vec{E}$.

(2) The work done by a constant force $\vec{F}$ on a particle undergoing a displacement $\vec{d}$ is

$$W = \vec{F} \cdot \vec{d}. \qquad (24\text{-}3)$$

(3) The electrostatic force and the electric field are related by the force equation $\vec{F} = q\vec{E}$, where here q is the charge of an electron ($= -1.6 \times 10^{-19}$ C).

Calculations: Substituting for $\vec{F}$ in Eq. 24-3 and taking the dot product yield

$$W = q\vec{E} \cdot \vec{d} = qEd \cos \theta, \qquad (24\text{-}4)$$

where θ is the angle between the directions of $\vec{E}$ and $\vec{d}$. The field $\vec{E}$ is directed downward and the displacement $\vec{d}$ is directed upward; so $\theta = 180°$. Substituting this and other data into Eq. 24-4, we find

$$W = (-1.6 \times 10^{-19}\ \text{C})(150\ \text{N/C})(520\ \text{m}) \cos 180°$$
$$= 1.2 \times 10^{-14}\ \text{J}.$$

Equation 24-1 then yields

$$\Delta U = -W = -1.2 \times 10^{-14}\ \text{J}. \qquad \text{(Answer)}$$

This result tells us that during the 520 m ascent, the electric potential energy of the electron *decreases* by 1.2×10^{-14} J.

Electric Potential

- The potential energy of a charged particle in an electric field depends on the charge magnitude
- Potential energy *per unit charge U/q* has a unique value at any point in an electric field
- ***Electric potential V*** (or simply ***potential***): the potential energy per unit charge at a point in an electric field

$$V = \frac{U}{q}.$$

- Electric potential is a *scalar* quantity
- The ***electric potential difference ΔV*** between any two points *i* and *f* in an electric field is equal to the difference in potential energy per unit charge between the two points:

$$\Delta V = V_f - V_i = \frac{U_f}{q} - \frac{U_i}{q} = \frac{\Delta U}{q} = -\frac{W}{q}$$

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- ΔV can be +ve, -ve, or zero, depending on the signs & magnitudes of q & W

- If $U_i = 0$ at ∞ → V_i = zero → the electric potential at any point in an electric field is $V = -\frac{W_\infty}{q}$
  - W_∞ is the work done by E to move q from ∞ → f

- The SI unit for potential is ***volt*** (**V**)
  1 volt = 1 joule / coulomb

- The E unit (N/C) is equivalent to V/m **?**
  (from work unit, J = V.C = N.m → N/C = V/m

- ***Electron-volt* (eV) unit:** the energy equal to the work required to move a single elementary charge e, (electron or proton), through a potential difference of exactly one volt
  - The magnitude of this work is $q\,\Delta V$
  - eV is the energy unit for measurements in atomic & subatomic scale
  - The relation between eV and joule:

$$1 \text{ eV} = e(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) = 1.60 \times 10^{-19} \text{ J.}$$

An Applied Work W_{app} Done by an Applied Force F_{app}

- If a charge q is moved from point i to point f by applying a force to it in an electric field
  → F_{app} does work W_{app} on the charge, and E does work W on it

- From the theorem of work–kinetic energy (the change in kinetic energy = work):

$$\Delta K = K_f - K_i = W_{\text{app}} + W.$$

- If the particle is stationary before & after the move
  → K_f & K_i = zero,
  → $W_{\text{app}} = -W.$
  the W_{app} by F_{app} = $-W$ done by E (if there is no change in K)

- The work done by F_{app} is related to the change in the potential energy of the particle during the move

$$\Delta U = U_f - U_i = W_{\text{app}}.$$

- W_{app} is related to the electric potential difference between the initial and final locations of the particle as

$$W_{\text{app}} = q\,\Delta V.$$

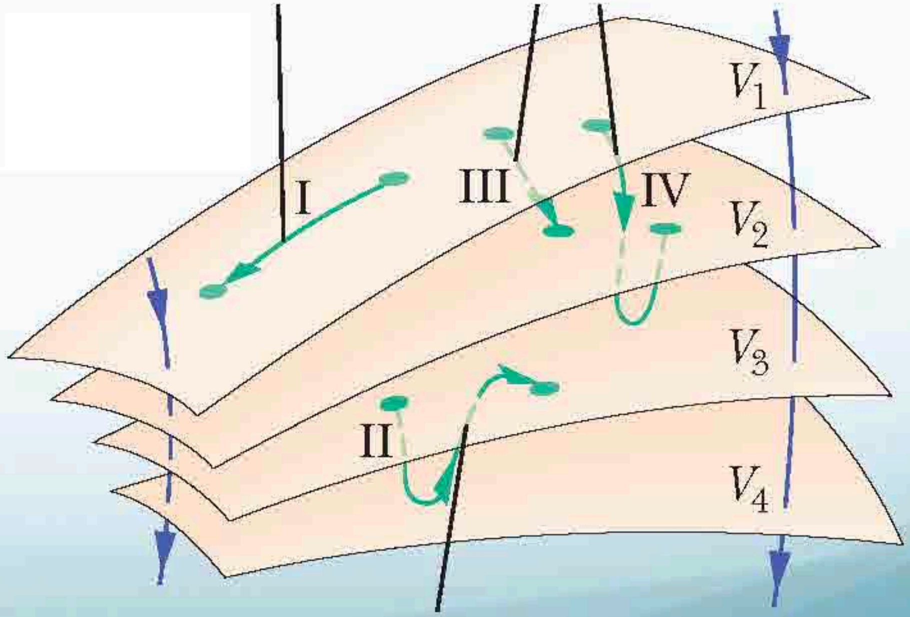
  - W_{app} can be +ve, -ve, or zero depending on the signs & magnitudes of q and ΔV

Equipotential Surfaces

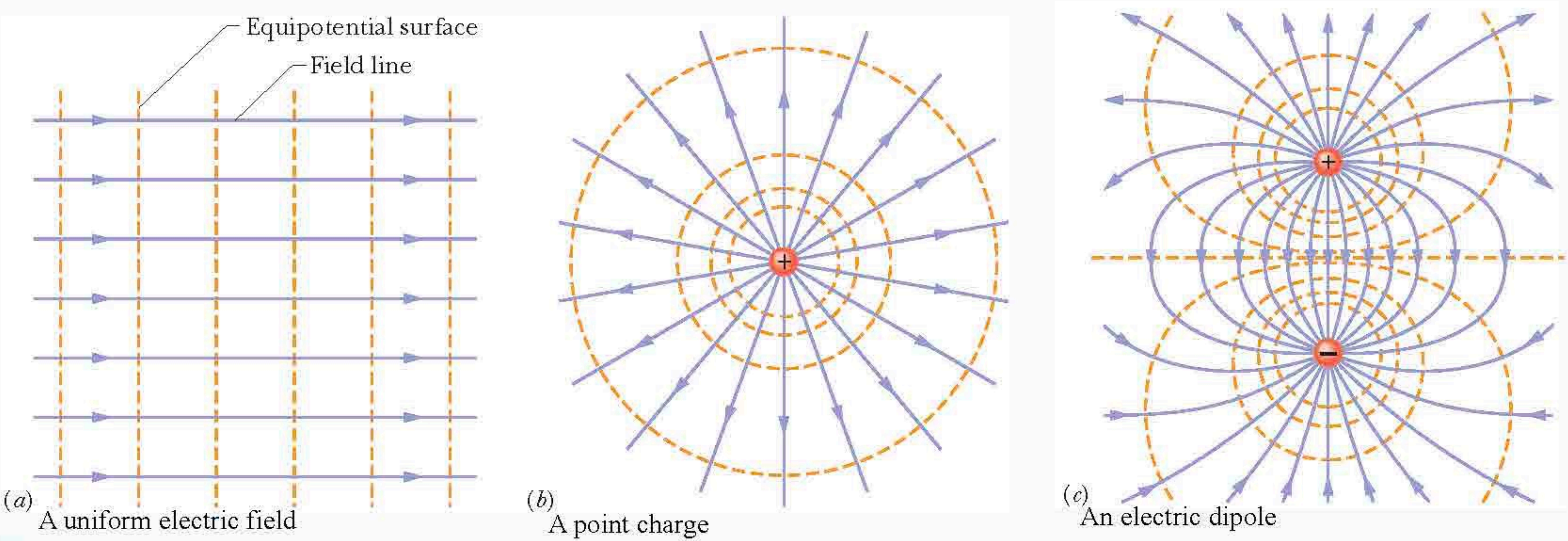
Equipotential surface: is a surface in which all points on it have the same electric potential

- Equipotential surface can be imaginary or real physical surface
- The potential difference between any points on the equipotential surface is zero
- For one equipotential surface $V_f = V_i \rightarrow W_{net} = 0$
- $W = 0$ for *any* path connecting points *i* and *f* on a given equipotential surface regardless of whether that path lies entirely on that surface.

- In the figure, there are 4 equipotential surfaces
  - For the paths I : $W_I = 0$ because $\Delta V = 0$
  - For path II: $W_{II} = 0$ because $\Delta V = 0$
  - For path III: $W_{III} = q\Delta V = q\,(V_2 - V_1)$
  - For path IV: $W_{IV} = q\Delta V = q\,(V_2 - V_1)$, $W_{IV} = W_{III}$



Examples for Equipotential Surfaces

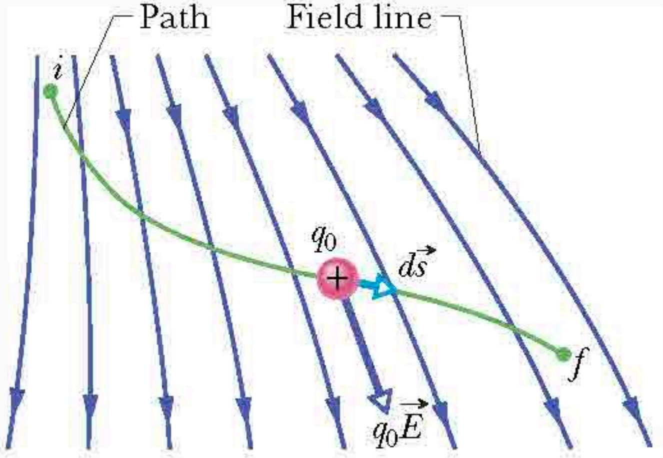


- For a uniform electric field: the equipotential surfaces are a family of planes perpendicular to the field lines
- For a point charge: the equipotential surfaces are a family of concentric spheres
- Equipotential surfaces are always perpendicular to electric field ***E***

Calculating the Potential from the Field

For $+q_0$ moves along the path from *i* to *f* in an electric field

- At any point on the path, $\boldsymbol{F} = q_0 \boldsymbol{E}$ on the charge as it moves through a differential displacement ***ds***



- The work dW on a particle by ***F*** during a displacement ***ds*** is:

$$dW = \vec{F}\cdot d\vec{s} \quad \rightarrow \quad dW = q_0\vec{E}\cdot d\vec{s}.$$

$$\rightarrow \quad W = q_0 \int_i^f \vec{E}\cdot d\vec{s}$$

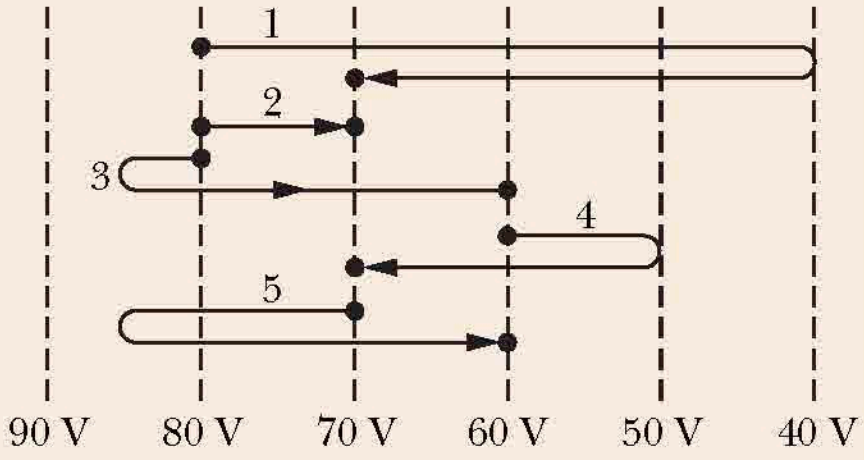
$$\rightarrow \quad V_f - V_i = -\int_i^f \vec{E}\cdot d\vec{s}$$

$$\Delta V = V_f - V_i = -\frac{W}{q}$$

- If $V_i = 0 \rightarrow$ $V = -\int_i^f \vec{E}\cdot d\vec{s}$
  - The equation gives *V* at any point *f relative to the zero potential* at point *i*
  - If $i = \infty \rightarrow$ the equation gives *V* at any point *f* relative to the zero potential at infinity

CHECKPOINT 3

The figure here shows a family of parallel equipotential surfaces (in cross section) and five paths along which we shall move an electron from one surface to another. (a) What is the direction of the electric field associated with the surfaces? (b) For each path, is the work we do positive, negative, or zero? (c) Rank the paths according to the work we do, greatest first.



(a) The electric field points *from* higher potential *to* lower potential

High V —— $\boldsymbol{E}$ ——→ Low V

→ $\boldsymbol{E}$ directed rightward;

(b) $W_{app} = q\,\Delta V$, $q = -e$

→ Paths 1, 2, 3, 5 have positive work; path 4 has negative work

(c) 3, then 1, 2, and 5tie, then 4

Sample Problem

(a) Figure 24-5*a* shows two points *i* and *f* in a uniform electric field $\vec{E}$. The points lie on the same electric field line (not shown) and are separated by a distance *d*. Find the potential difference $V_f - V_i$ by moving a positive test charge q_0 from *i* to *f* along the path shown, which is parallel to the field direction.

$$\vec{E} \cdot d\vec{s} = E\,ds\cos\theta = E\,ds. \qquad (24\text{-}20)$$

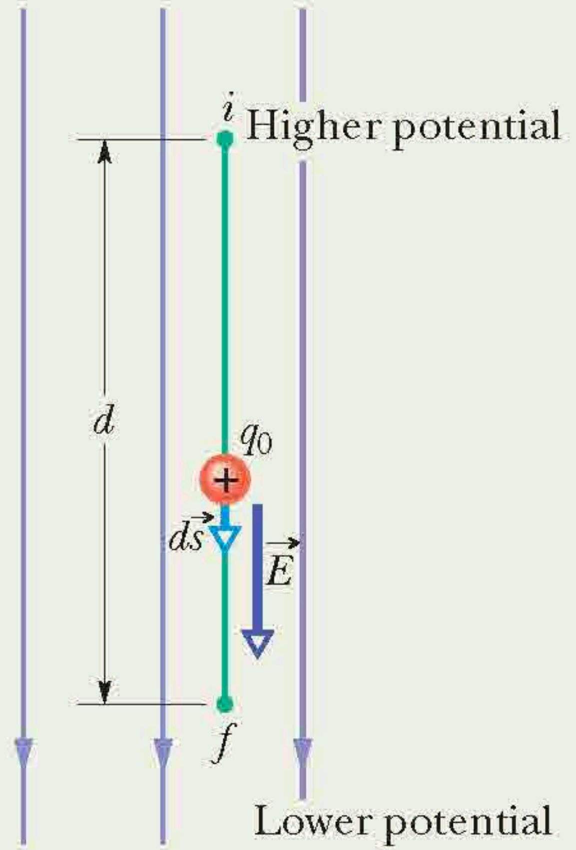
Equations 24-18 and 24-20 then give us

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s} = -\int_i^f E\,ds. \qquad (24\text{-}21)$$

Since the field is uniform, *E* is constant over the path and can be moved outside the integral, giving us

$$V_f - V_i = -E\int_i^f ds = -Ed, \qquad \text{(Answer)}$$

The electric field points *from* higher potential *to* lower potential



(*a*)

(b) Now find the potential difference $V_f - V_i$ by moving the positive test charge q_0 from i to f along the path icf shown in Fig. 24-5b.

Calculations: The Key Idea of (a) applies here too, except now we move the test charge along a path that consists of two lines: ic and cf. At all points along line ic, the displacement $d\vec{s}$ of the test charge is perpendicular to $\vec{E}$. Thus, the angle θ between $\vec{E}$ and $d\vec{s}$ is 90°, and the dot product $\vec{E} \cdot d\vec{s}$ is 0. Equation 24-18 then tells us that points i and c are at the same potential: $V_c - V_i = 0$.

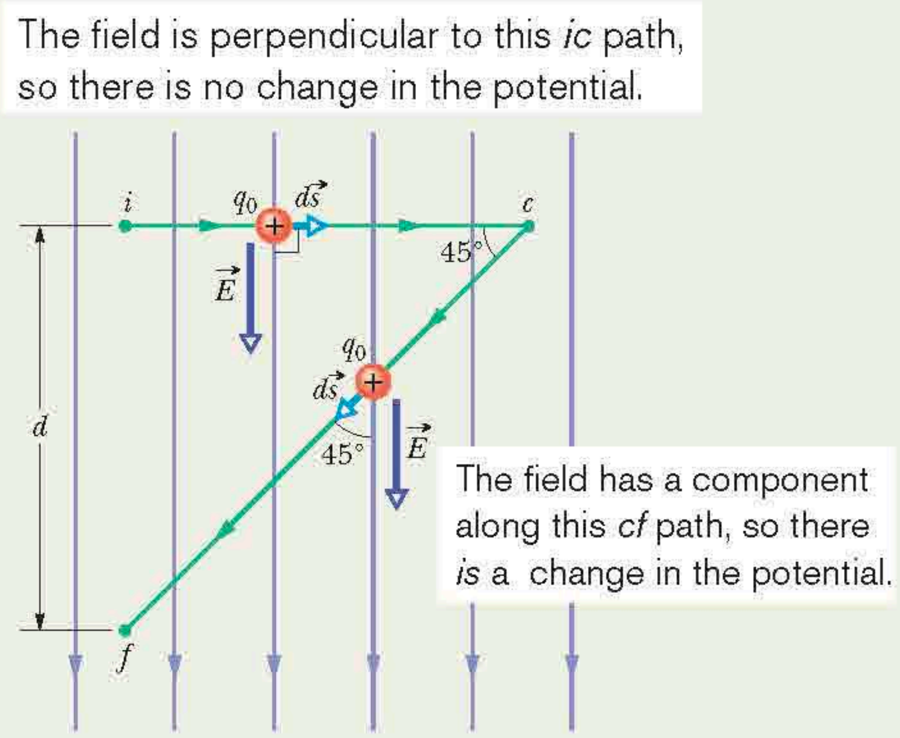
For line cf we have $\theta = 45°$ and, from Eq. 24-18,

$$V_f - V_i = -\int_c^f \vec{E} \cdot d\vec{s} = -\int_c^f E(\cos 45°)\, ds$$

$$= -E(\cos 45°) \int_c^f ds.$$

The integral in this equation is just the length of line cf; from Fig. 24-5b, that length is $d/\cos 45°$. Thus,

$$V_f - V_i = -E(\cos 45°) \frac{d}{\cos 45°} = -Ed. \quad \text{(Answer)}$$



(*b*)

Potential Due to a Point Charge

To find V at point P that lies in a distance R from $+q$

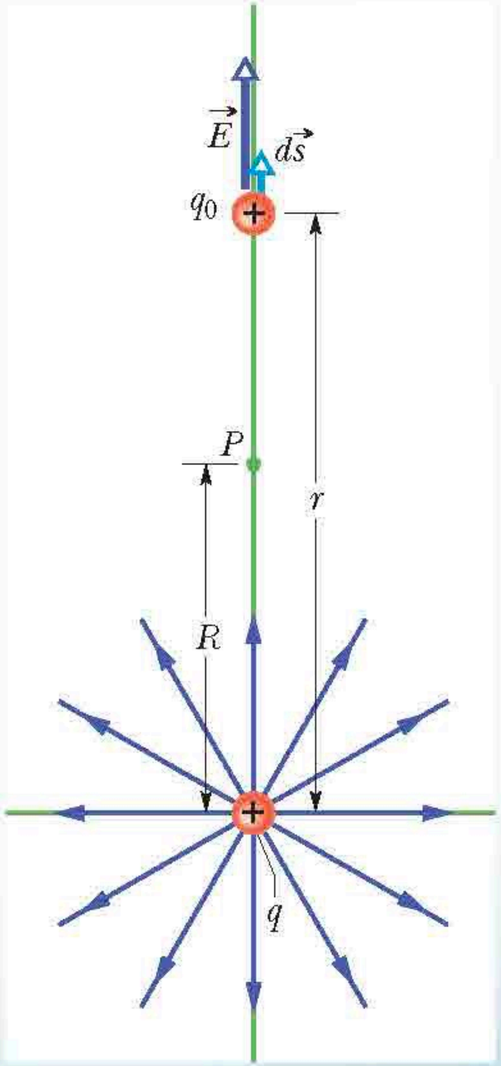
- We imagine that we move +ve test charge q_0 from point P to infinity
- The dot product is $\vec{E} \cdot d\vec{s} = E \cos\theta \, ds.$
- Where ***E*** directed radially outward from the fixed particle, ***ds*** has same direction as ***E*** → $\theta = 0, \cos\theta = 1$
- Because the path is radial → we write *ds* as *dr*. Then, substituting the limits R & ∞:

$$V_f - V_i = -\int_R^\infty E \, dr.$$

- We set $V_f = 0 \text{ (at } \infty)$ and $V_i = V \text{ (at } R)$, and $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$

$$0 - V = -\frac{q}{4\pi\varepsilon_0}\int_R^\infty \frac{1}{r^2}\,dr = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]_R^\infty$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$



- This equation also gives V either *outside or on the external surface of* a spherically symmetric charge distribution

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential.

Examples:

1. **Calculate the electric potential at a distance of 5cm from a point charge of 2.5nC.**

$$V = \frac{kq}{r} = \frac{9\times10^{9}\times2.5\times10^{-9}}{0.05} = 450V$$

2. **A point charge produces an electric field of 180N/C at 2cm. Calculate the electric potential at 4cm from the charge.**

To find V at any distance from a charge, we need to calculate q

The electric field due to a point charge is given by: $E = \frac{kq}{r^2} \longrightarrow q = \frac{Er^2}{k} = \frac{180\times0.02^2}{9\times10^9} = 8\times10^{-12}C$

The electric potential is: $V = \frac{kq}{r} = \frac{9\times10^9\times8\times10^{-12}}{0.04} = 1.8V$

Potential Due to a Group of Point Charges

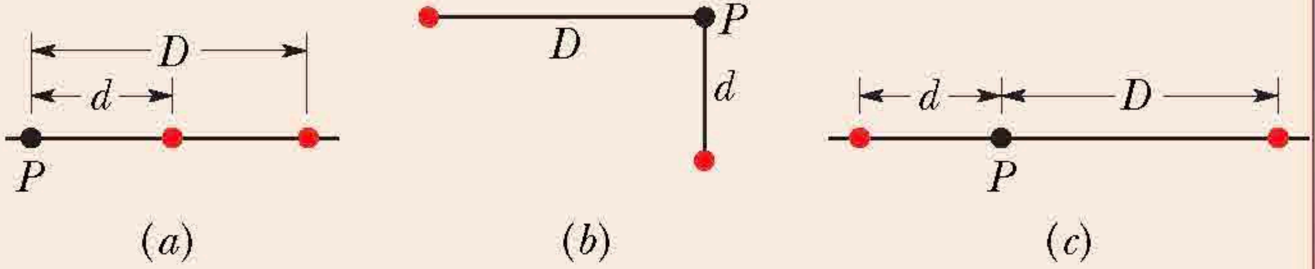
We can find the net potential at a point due to a group of point charges using the superposition principle

$$V = \sum_{i=1}^{n} V_i = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i} \qquad (n \text{ point charges}).$$

The sum here is an *algebraic sum*, not a vector sum like the sum used to calculate ***E*** for a group of point charges

CHECKPOINT 4

The figure here shows three arrangements of two protons. Rank the arrangements according to the net electric potential produced at point P by the protons, greatest first.



All tie

Sample Problem

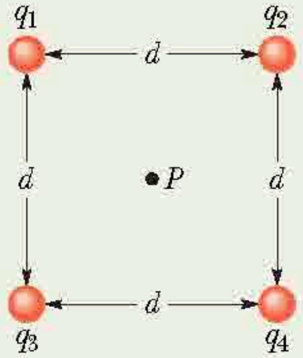
Net potential of several charged particles

What is the electric potential at point P, located at the center of the square of point charges shown in Fig. 24-8a? The distance d is 1.3 m, and the charges are

$$q_1 = +12\text{ nC}, \qquad q_3 = +31\text{ nC},$$
$$q_2 = -24\text{ nC}, \qquad q_4 = +17\text{ nC}.$$

KEY IDEA

The electric potential V at point P is the algebraic sum of the electric potentials contributed by the four point charges.



(a)

Fig. 24-8 (a) Four point charges are held fixed at the corners of a square.

(Because electric potential is a scalar, the orientations of the point charges do not matter.)

Calculations: From Eq. 24-27, we have

$$V = \sum_{i=1}^{4} V_i = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r} + \frac{q_2}{r} + \frac{q_3}{r} + \frac{q_4}{r}\right).$$

The distance r is $d/\sqrt{2}$, which is 0.919 m, and the sum of the charges is

$$q_1 + q_2 + q_3 + q_4 = (12 - 24 + 31 + 17) \times 10^{-9}\text{ C}$$
$$= 36 \times 10^{-9}\text{ C}.$$

Thus,
$$V = \frac{(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(36 \times 10^{-9}\text{ C})}{0.919\text{ m}}$$
$$\approx 350\text{ V}. \qquad \text{(Answer)}$$

Sample Problem

Potential is not a vector, orientation is irrelevant

(a) In Fig. 24-9*a*, 12 electrons (of charge $-e$) are equally spaced and fixed around a circle of radius R. Relative to $V = 0$ at infinity, what are the electric potential and electric field at the center C of the circle due to these electrons?

KEY IDEAS

(1) The electric potential V at C is the algebraic sum of the electric potentials contributed by all the electrons. (Because electric potential is a scalar, the orientations of the electrons do not matter.) (2) The electric field at C is a vector quantity and thus the orientation of the electrons *is* important.

Calculations: Because the electrons all have the same negative charge $-e$ and are all the same distance R from C, Eq. 24-27 gives us

$$V = -12 \frac{1}{4\pi\varepsilon_0} \frac{e}{R}. \qquad \text{(Answer)} \qquad (24\text{-}28)$$

Because of the symmetry of the arrangement in Fig. 24-9*a*, the electric field vector at C due to any given electron is canceled by the field vector due to the electron that is diametrically opposite it. Thus, at C,

$$\vec{E} = 0. \qquad \text{(Answer)}$$

Potential is a scalar and orientation is irrelevant.

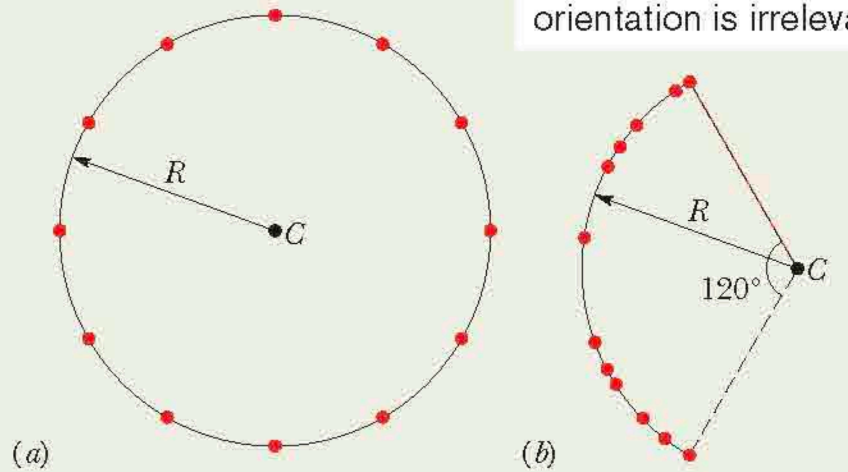


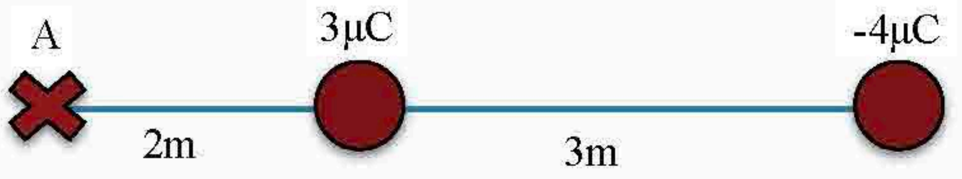
Fig. 24-9 (*a*) Twelve electrons uniformly spaced around a circle. (*b*) The electrons nonuniformly spaced along an arc of the original circle.

(b) If the electrons are moved along the circle until they are nonuniformly spaced over a 120° arc (Fig. 24-9*b*), what then is the potential at C? How does the electric field at C change (if at all)?

Reasoning: The potential is still given by Eq. 24-28, because the distance between C and each electron is unchanged and orientation is irrelevant. The electric field is no longer zero, however, because the arrangement is no longer symmetric. A net field is now directed toward the charge distribution.

Examples:

3. As shown in the arrangement below, calculate the electric potential at point A. Calculate the work needed to bring a charge of 6nC from infinity to point A.



The electric potential:

$$V = V_1 + V_2 = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = 9\times10^9(\frac{3\times10^{-6}}{2} + \frac{-4\times10^{-6}}{5}) = 6300V$$

The work needed:

$$W = -qV = -6\times10^{-9}\times6300 = -3.78\times10^{-5}J = \frac{-3.78\times10^{-5}}{1.6\times10^{-19}}eV = -2.36\times10^{14}eV$$

Potential Due to an Electric Dipole

To find the potential due to an electric dipole at point P

- $+q$ (at distance $r_{(+)}$) gives $V_{(+)}$
- $-q$ (at distance $r_{(-)}$) gives potential $V_{(-)}$

$$V = \sum_{i=1}^{2} V_i = V_{(+)} + V_{(-)} = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_{(+)}} + \frac{-q}{r_{(-)}}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{r_{(-)} - r_{(+)}}{r_{(-)}r_{(+)}}.$$

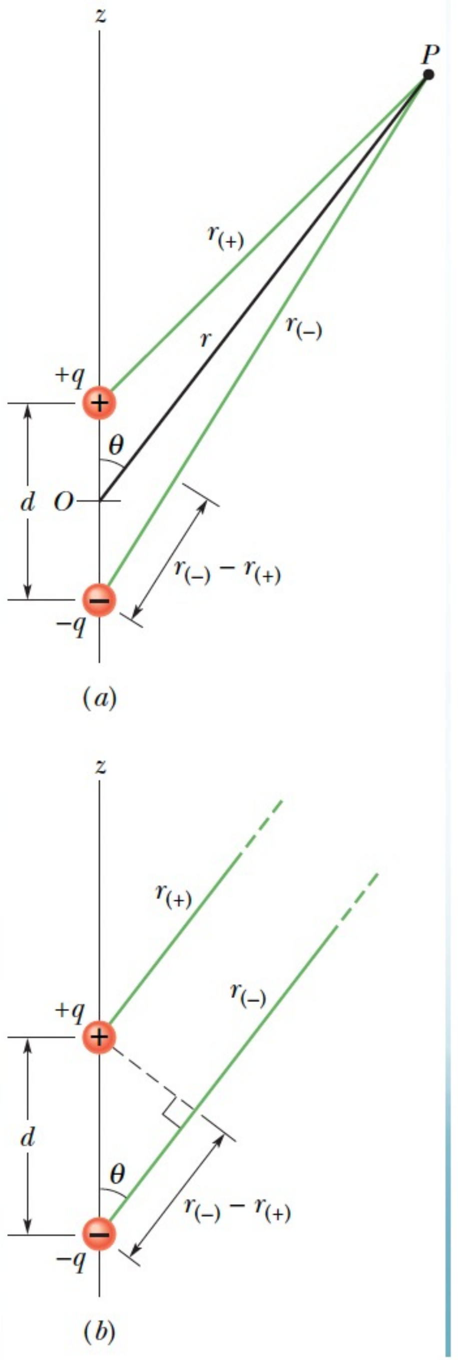
- Where d is the distance between the charges, θ is measured from the dipole axis
- Far from the dipole, $r >> d \rightarrow$ $r_{(-)} - r_{(+)} \approx d\cos\theta$ and $r_{(-)}r_{(+)} \approx r^2$

$$V = \frac{q}{4\pi\varepsilon_0}\frac{d\cos\theta}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2} \qquad \text{(electric dipole).}$$

Where p is the magnitude of the electric dipole moment
The vector $\boldsymbol{p}$ is directed along the dipole axis, from the –ve to +ve charge

θ is measured from the direction of $\boldsymbol{p}$

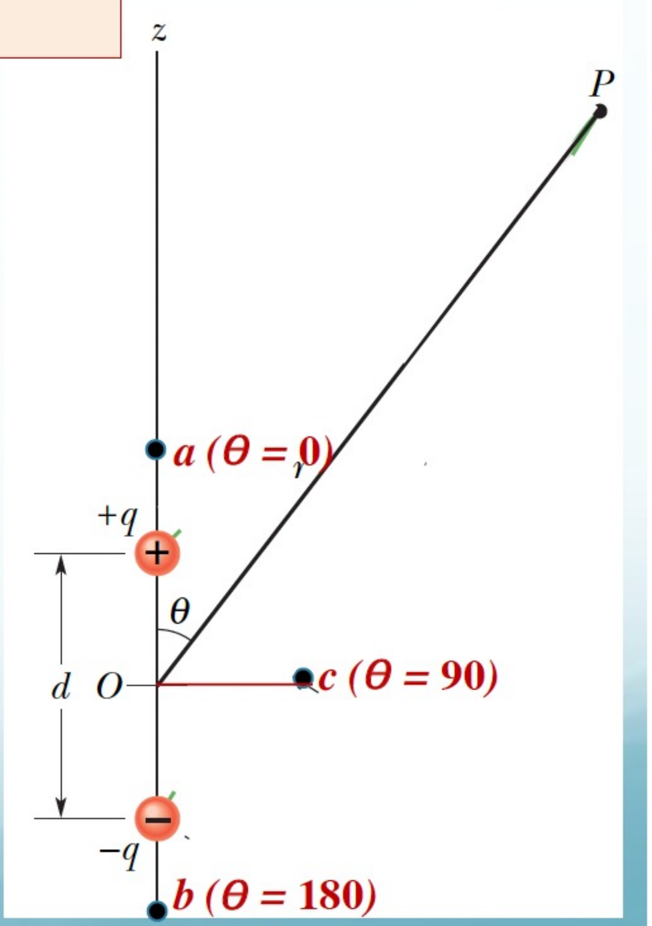


CHECKPOINT 5

Suppose that three points are set at equal (large) distances r from the center of the dipole in Fig. 24-10: Point a is on the dipole axis above the positive charge, point b is on the axis below the negative charge, and point c is on a perpendicular bisector through the line connecting the two charges. Rank the points according to the electric potential of the dipole there, greatest (most positive) first.

$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

a, c (zero), b



Examples:

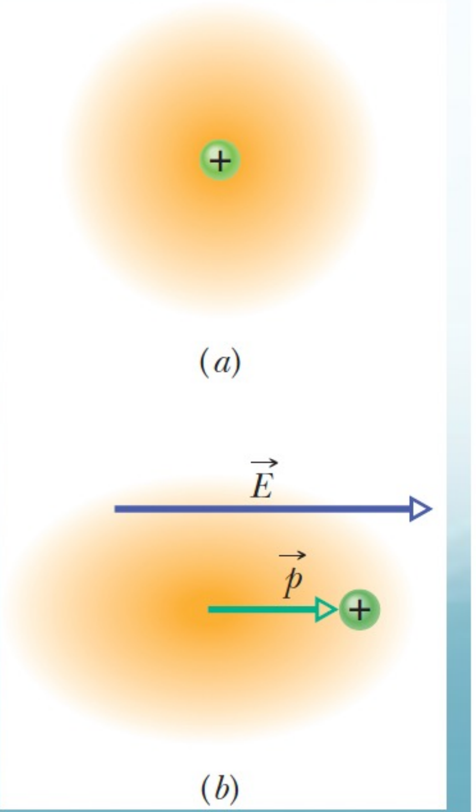
4. Two equal and opposite charges 6 & -6µC are separated by a distance of 2cm. What is the electric potential at a point *P* above the positive charge and of 3cm away from the center of the dipole axis?

$$V = \frac{kp\cos\theta}{r^2} = \frac{kqd\cos 0}{r^2} = \frac{9\times 10^{9}\times 6\times 10^{-6}\times 0.02}{0.03^2} = 1200000V$$

Induced Dipole Moment

- Many molecules, such as water, have *permanent* ***p***

- For *nonpolar molecules* & *isolated atoms*, the centers of +ve & -ve charges coincide
  → ***p*** = 0
- If nonpolar molecules & isolated atoms are placed in ***E***
  - → the electron orbits is distorted and separates the centers of +ve & -ve charge
  - Because the electrons has $-q$ → electrons are shifted in a direction opposite ***E*** **?**
  - →***p*** is produced in the direction of ***E***
  - This dipole moment is said to be ***induced*** by the field
  - The atom or molecule is then said to be ***polarized*** by the field (means it has +ve & –ve side)
- When the field is <u>removed</u>, the induced dipole moment and the polarization <u>disappear</u>



Potential of a Charged Isolated Conductor

- The potential of a moving charge from i to f in an electric field is:

$$V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}.$$

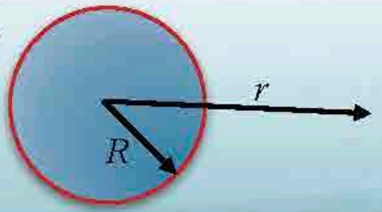
- Within a conductor, $E = 0$ for all points i & f in the conductor → $V_i = V_f$

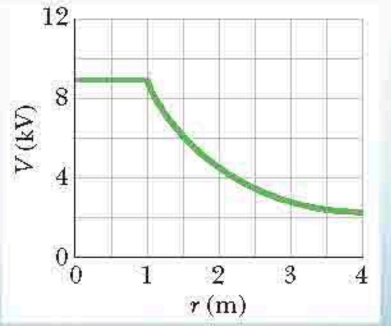
An excess charge placed on an isolated conductor will distribute itself on the surface of that conductor so that all points of the conductor—whether on the surface or inside—come to the same potential. This is true even if the conductor has an internal cavity and even if that cavity contains a net charge.

- For a charged conducting sphere of radius R, the electric potential at any distance is given by:

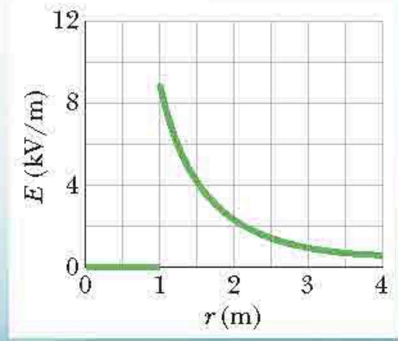
$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

  - Inside, on the surface of sphere: $V = \frac{kq}{R}$
  - Outside the sphere: $V = \frac{kq}{r}$





A plot of $V(r)$ both inside & outside a charged spherical shell of radius 1.0 m.



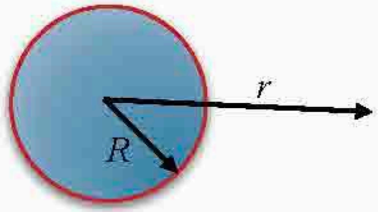
A plot of $E(r)$ for the same shell.

Examples:

7. **A metallic sphere of radius 10cm has a charge of 5μC. What is the electric potential at:**

   I. **5cm**

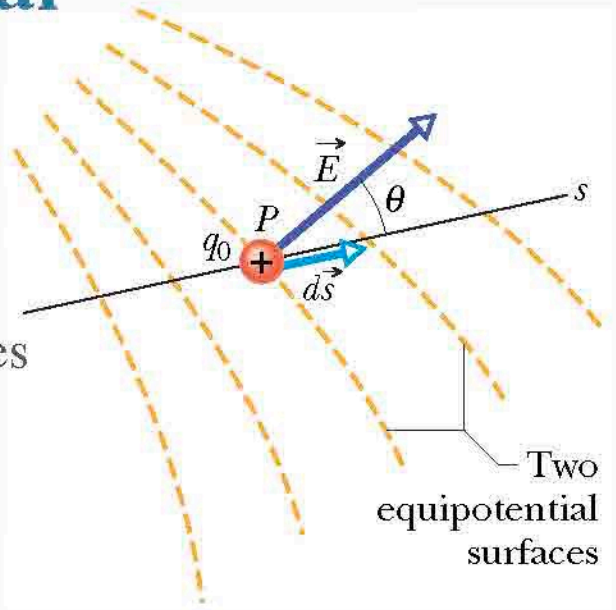
$$V = \frac{kq}{R} = \frac{9\times10^{9}\times5\times10^{-6}}{0.1} = 450000V$$



   II. **15cm**

$$V = \frac{kq}{r} = \frac{9\times10^{9}\times5\times10^{-6}}{0.15} = 300000V$$

Calculating the Field from the Potential



- The figure shows cross sections of equipotential surfaces
  The potential differences between 2 surfaces is dV
  E at point P is perpendicular to the equipotential surface through P

- A test charge $+q_0$ moves through a displacement ***ds*** between 2 equipotential surfaces
  - The work due E is $-q_0\,dV$. The work is also $(q_0\vec{E})\cdot d\vec{s}$, or $q_0E(\cos\theta)\,ds$.

    → $-q_0\,dV = q_0E(\cos\theta)\,ds$. → $E\cos\theta = -\dfrac{dV}{ds}$

    → $$E_s = -\frac{\partial V}{\partial s}.$$ E_s: component of ***E*** in the direction of ***ds***

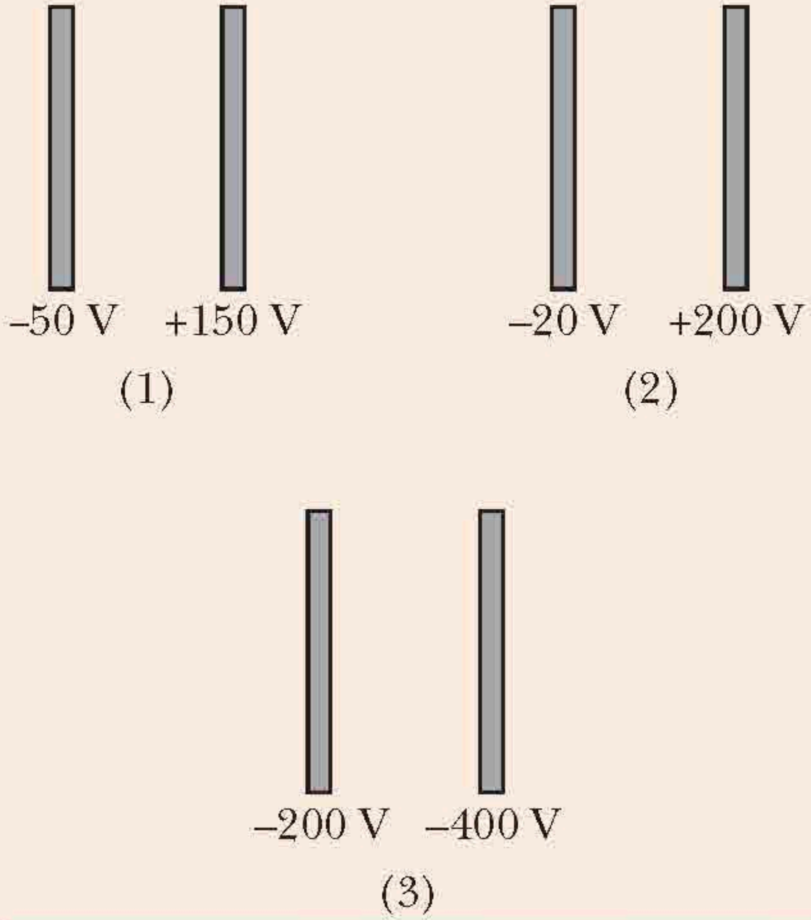
  - The x, y, and z components of ***E*** at any point are $$E_x = -\frac{\partial V}{\partial x};\qquad E_y = -\frac{\partial V}{\partial y};\qquad E_z = -\frac{\partial V}{\partial z}$$

The component of $\vec{E}$ in any direction is the negative of the rate at which the electric potential changes with distance in that direction.

- If E is uniform $$E = -\frac{\Delta V}{\Delta s},$$
  where s is perpendicular to the equipotential surfaces

CHECKPOINT 6

The figure shows three pairs of parallel plates with the same separation, and the electric potential of each plate. The electric field between the plates is uniform and perpendicular to the plates. (a) Rank the pairs according to the magnitude of the electric field between the plates, greatest first. (b) For which pair is the electric field pointing rightward? (c) If an electron is released midway between the third pair of plates, does it remain there, move rightward at constant speed, move leftward at constant speed, accelerate rightward, or accelerate leftward?



(a) $E = -\frac{\Delta V}{\Delta s}$,

(1) $E.\Delta s$ = -(-50-150) = 200
(2) $E.\Delta s$ = -(-20-200) = 220
(3) $E.\Delta s$ = -(-400+200) = 200

→2, then1 and 3 tie;

(b) 3;
(c) accelerate leftward

Sample Problem

Finding the field from the potential

The electric potential at any point on the central axis of a uniformly charged disk is given by Eq. 24-37,

$$V = \frac{\sigma}{2\varepsilon_0}(\sqrt{z^2 + R^2} - z).$$

Starting with this expression, derive an expression for the electric field at any point on the axis of the disk.

KEY IDEAS

We want the electric field $\vec{E}$ as a function of distance z along the axis of the disk. For any value of z, the direction of $\vec{E}$ must be along that axis because the disk has circular symmetry about that axis. Thus, we want the component E_z of $\vec{E}$ in the direction of z. This component is the negative of the rate at which the electric potential changes with distance z.

Calculation: Thus, from the last of Eqs. 24-41, we can write

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\varepsilon_0}\frac{d}{dz}(\sqrt{z^2 + R^2} - z)$$

$$= \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right). \qquad \text{(Answer)}$$

This is the same expression that we derived in Section 22-7 by integration, using Coulomb's law.

Examples:

6. **At a certain region, the electric potential is found to be $V(x,y,z) = 3x^2y + z$. Calculate the electric field at point (1,0,1).**

$$E_x = -\frac{\partial V}{\partial x}; \qquad E_y = -\frac{\partial V}{\partial y}; \qquad E_z = -\frac{\partial V}{\partial z}.$$

$E_x = -6xy$, $E_y = -3x^2$, $E_z = -1$

At point (1,0,1) → $E_x = 0$, $E_y = -3$, $E_z = -1$

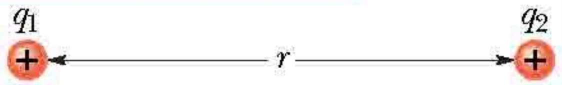
→ $\vec{E} = -3\hat{j} - \hat{k}$

Electric Potential Energy of a System of Point Charges

- Previously we defined the electric potential energy U for a point charge as an electrostatic force does work on it
- Here we want to find the electric potential energy of a *system* of charges due to the electric field produced *by* those <u>same</u> charges

> The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bringing each charge in from an infinite distance.

- To find U for a system of 2 charges q_1 & q_2, separated by a distance r



  - When we bring q_1 from ∞ to a place → we do no work? because no electrostatic force acts on q_1
  - When we bring q_2 from ∞ to a place → we must do work? because q_1 exerts an electrostatic force on q_2 during the move, → $W = q_2V$, where V is the potential on q_2 by q_1,

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r}.$$

- The electric potential energy of the <u>pair</u> of point charges: $U = W = q_2V = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$
  - If q_1 & q_2 have same sign → +ve work needed to push them together against their mutual <u>repulsion</u> → U of the system is +ve
  - If q_1 & q_2 have opposite signs →–ve work needed against their mutual <u>attraction</u> to bring them together → U of the system is -ve

Sample Problem

Potential energy of a system of three charged particles

Figure 24-16 shows three point charges held in fixed positions by forces that are not shown. What is the electric potential energy U of this system of charges? Assume that $d = 12$ cm and that

$$q_1 = +q, \quad q_2 = -4q, \quad \text{and} \quad q_3 = +2q,$$

in which $q = 150$ nC.

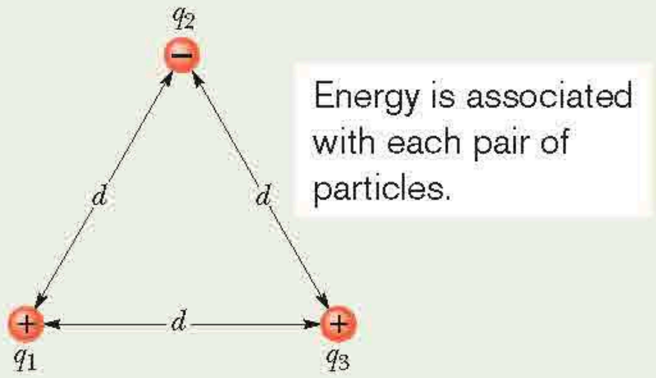


Fig. 24-16 Three charges are fixed at the vertices of an equilateral triangle. What is the electric potential energy of the system?

KEY IDEA

The potential energy U of the system is equal to the work we must do to assemble the system, bringing in each charge from an infinite distance.

$$U = U_{12} + U_{13} + U_{23}$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{(+q)(-4q)}{d} + \frac{(+q)(+2q)}{d} + \frac{(-4q)(+2q)}{d}\right)$$

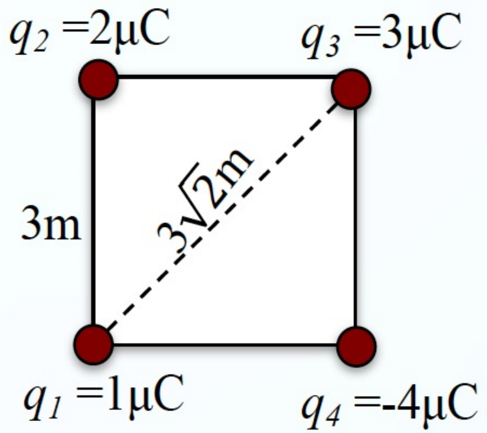
$$= -\frac{10q^2}{4\pi\varepsilon_0 d}$$

$$= -\frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(10)(150 \times 10^{-9}\ \text{C})^2}{0.12\ \text{m}}$$

$$= -1.7 \times 10^{-2}\ \text{J} = -17\ \text{mJ}. \qquad \text{(Answer)}$$

Examples:

5. Four charges 1, 2, 3 & 4 are located at the corners of a square of side 3m as shown in the figure. Calculate the electric potential energy of the system.



$$U = \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_3q_4}{r_{34}} + \frac{kq_4q_1}{r_{41}} + \frac{kq_1q_3}{r_{13}} + \frac{kq_2q_4}{r_{24}}$$

$$= \frac{k}{r}(q_1q_2 + q_2q_3 + q_3q_4 + q_4q_1) + \frac{k}{\sqrt{2}r}(q_1q_3 + q_2q_4)$$

$$= \frac{9\times10^9}{3}\ (2 + 6 - 12 - 4)\times10^{-6} + \frac{9\times10^9}{3\sqrt{2}}\ (3 - 8)\times10^{-6}$$

$$= -3\times10^3\times8 - \frac{3\times10^3\times5}{\sqrt{2}} = -24\times10^3 - \frac{15\times10^3}{\sqrt{2}} = -34.7\text{k}J$$

•**2** The electric potential difference between the ground and a cloud in a particular thunderstorm is 1.2×10^9 V. In the unit electron-volts, what is the magnitude of the change in the electric potential energy of an electron that moves between the ground and the cloud?

•**5** SSM An infinite nonconducting sheet has a surface charge density $\sigma = 0.10\ \mu C/m^2$ on one side. How far apart are equipotential surfaces whose potentials differ by 50 V?

•**12** As a space shuttle moves through the dilute ionized gas of Earth's ionosphere, the shuttle's potential is typically changed by -1.0 V during one revolution. Assuming the shuttle is a sphere of radius 10 m, estimate the amount of charge it collects.

•**13** What are (a) the charge and (b) the charge density on the surface of a conducting sphere of radius 0.15 m whose potential is 200 V (with $V = 0$ at infinity)?

•**21** ILW The ammonia molecule NH_3 has a permanent electric dipole moment equal to 1.47 D, where 1 D = 1 debye unit = 3.34×10^{-30} C·m. Calculate the electric potential due to an ammonia molecule at a point 52.0 nm away along the axis of the dipole. (Set $V = 0$ at infinity.)

•**35** The electric potential at points in an xy plane is given by $V = (2.0\ V/m^2)x^2 - (3.0\ V/m^2)y^2$. In unit-vector notation, what is the electric field at the point (3.0 m, 2.0 m)?

•**42** (a) What is the electric potential energy of two electrons separated by 2.00 nm? (b) If the separation increases, does the potential energy increase or decrease?

•**65** SSM What is the excess charge on a conducting sphere of radius $r = 0.15$ m if the potential of the sphere is 1500 V and $V = 0$ at infinity?