General Physics 2
SCPH 211

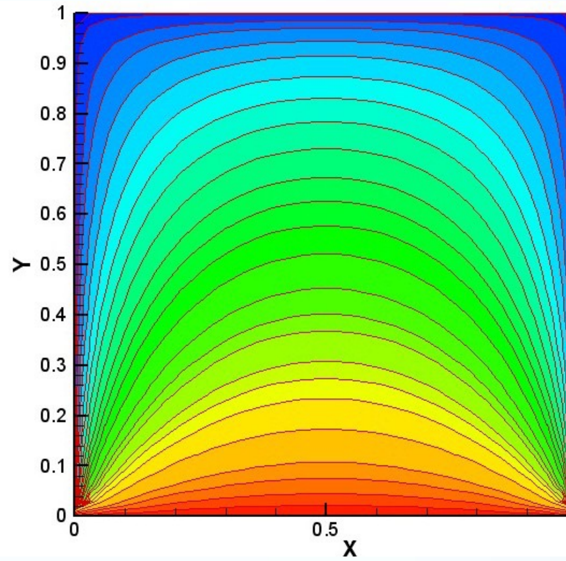
Chapter 22
Electric Fields

This dramatic photograph captures a lightning bolt striking a tree near some rural homes. Lightning is associated with very strong electric fields in the atmosphere.

Outline:

- Electric Field
- Electric Field Lines
- The Electric Field Due to
  - Point Charge
  - Line of Charge
  - Charged Disk
  - A Point Charge in an Electric Field

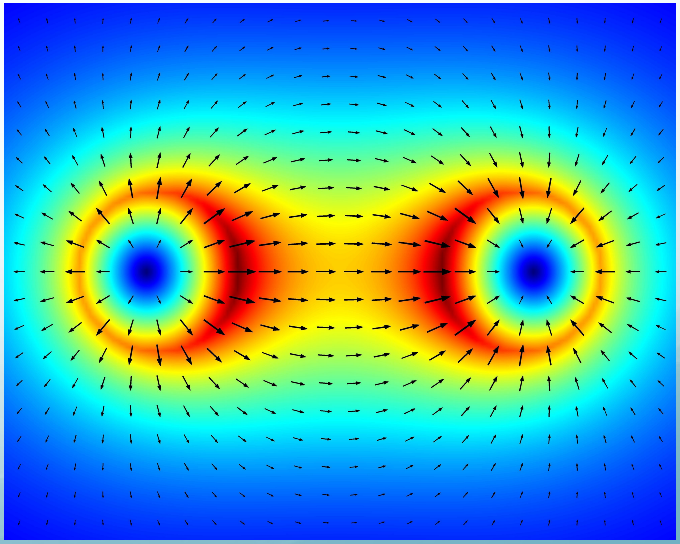
Electric Fields



How does q_1 "know" the presence of q_2?
Answer: each charge creates *electric field*

- *Field:* distribution of a quantity in space
  Example: temperature (measured by thermometer)
  à the distribution of temperatures in a region is a *temperature field* (*scalar fields*)

- The ***electric field*****:** is a region where electric charges can interact to each other
  - It is a vector field
  - It is a distribution of vectors for each point in the region around a charged object

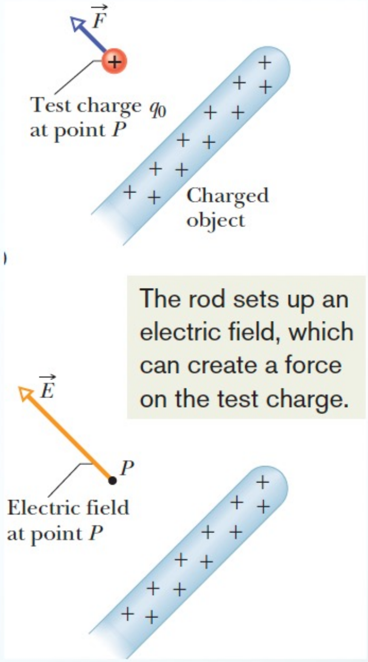


Electric Fields

How electric field is defined at a point *P* near a charged object?

1. Place +ve *test charge* q_0, at that point ***P***
2. Measure the electrostatic force $\vec{F}$ that acts on q_0
3. The electric field $\vec{E}$ at point P due to the charged object is:

$$\vec{E} = \frac{\vec{F}}{q_0} \qquad \text{(electric field).}$$



- The magnitude of ***E*** is $E = F/q_0$
  The direction of ***E*** is the direction of ***F***
- ***E*** represented with a vector whose tail is at P
- The SI unit for ***E*** is the newton per coulomb (N/C).

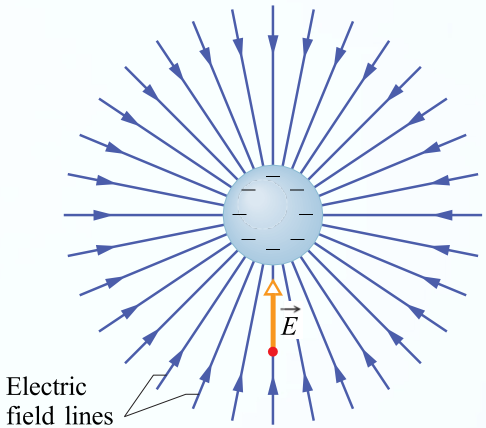
Electric Field Lines

- *Electric field lines:* lines of force around a charge
  Importance of electric field lines: to visualize patterns of electric fields

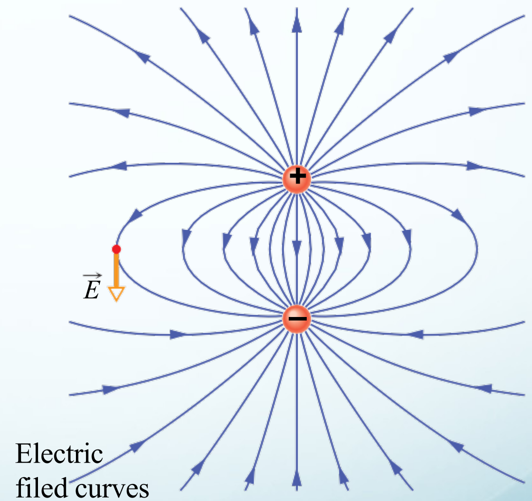
- **The relation between the field lines & electric field vectors *E*:**
  - The direction of ***E***: straight for a straight line or tangent for a curved field line
  - The magnitude of ***E***: proportional to the number of lines per unit area
    à *E* is large when lines are close together and small when they are far apart

- The magnitude of *E* decreases with distance from the charged object

- If the charged object has +ve charge
  à ***E*** directed away from the object

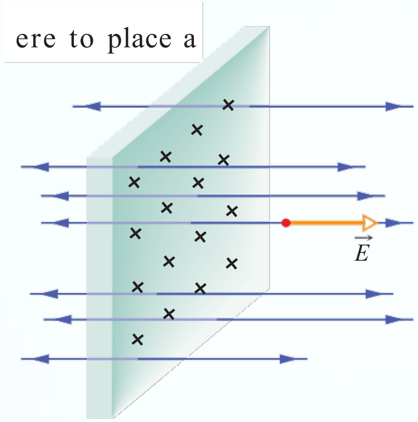


E is straight

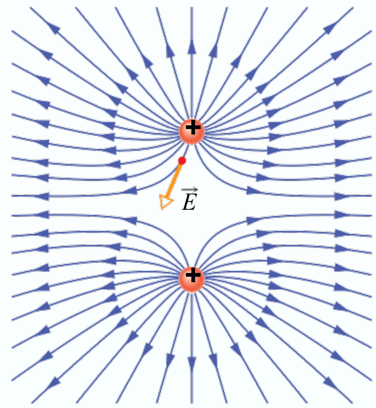


E is tangent

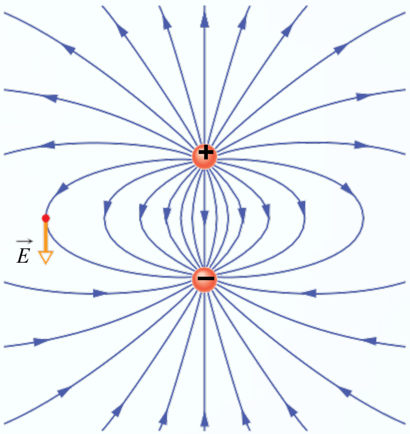
Types Electric Field Lines



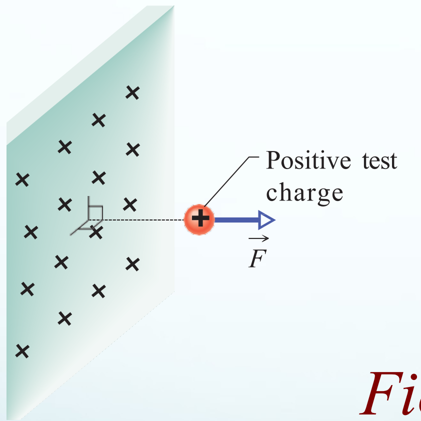
Uniform ***E***



Nonuniform ***E***



Electric dipole



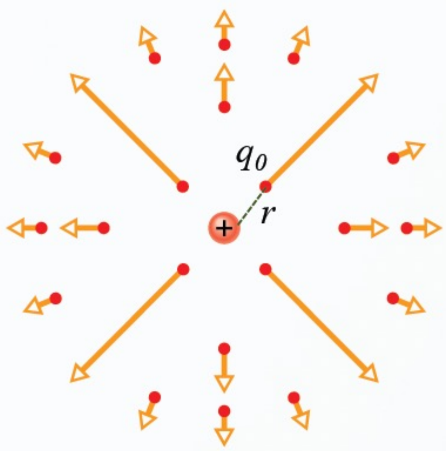
Field lines <u>never</u> cross

Electric field lines extend away from positive charge (where they originate) and toward negative charge (where they terminate).

The Electric Field Due to a Point Charge

- We put a +ve test charge q_0 at a distance r from the point charge q
  - The electrostatic force acting on q_0 is:

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q q_0}{r^2} \hat{r}$$



  - The electric field vector is:

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \hat{r} \qquad \text{(point charge)}.$$

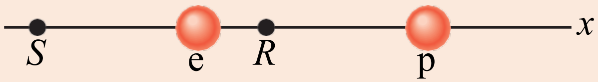
  - The direction of ***E*** is the same as ***F*** on the +ve test charge:
    If q is +ve → away from the point charge q
    If q is –ve → toward the point charge q

- The net (resultant) ***E*** due to more than one point charge

$$\vec{F}_0 = \vec{F}_{01} + \vec{F}_{02} + \cdots + \vec{F}_{0n}.$$

$$\vec{E} = \frac{\vec{F}_0}{q_0} = \frac{\vec{F}_{01}}{q_0} + \frac{\vec{F}_{02}}{q_0} + \cdots + \frac{\vec{F}_{0n}}{q_0} = \vec{E}_1 + \vec{E}_2 + \cdots + \vec{E}_n.$$

CHECKPOINT 1



The figure here shows a proton p and an electron e on an x axis. What is the direction of the electric field due to the electron at (a) point S and (b) point R? What is the direction of the net electric field at (c) point R and (d) point S?

(a) rightward;
(b) leftward;
(c) leftward;
(d) rightward

Sample Problem

Figure 22-7*a* shows three particles with charges $q_1 = +2Q$, $q_2 = -2Q$, and $q_3 = -4Q$, each a distance d from the origin. What net electric field $\vec{E}$ is produced at the origin?

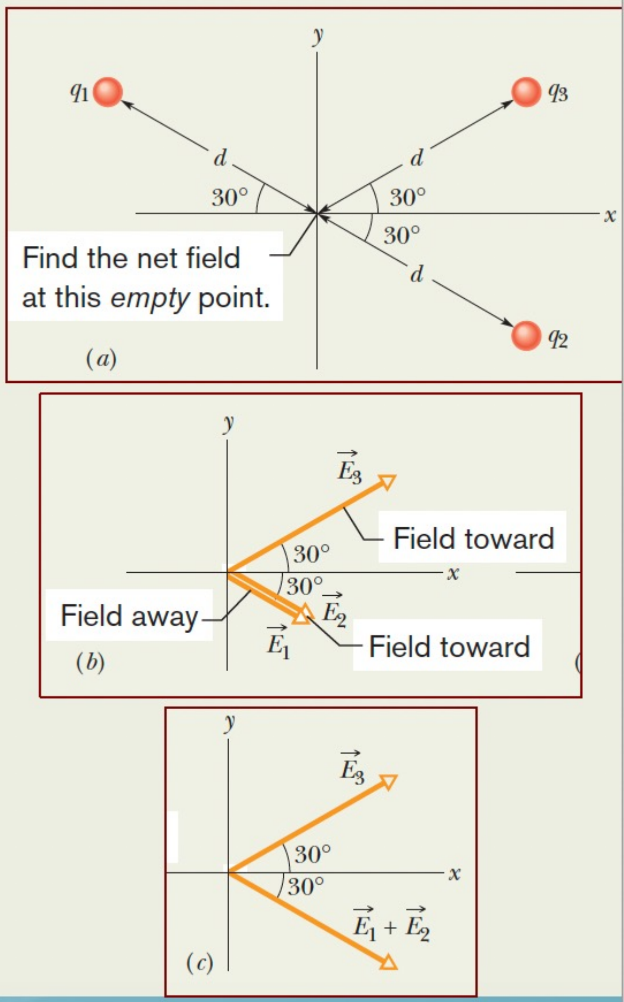
$$E_1 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2}, \quad E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} \quad \text{and} \quad E_3 = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}.$$

- Because q_1 is a +ve → $\boldsymbol{E_1}$ away from it, and because q_2 and q_3 are -ve → $\boldsymbol{E_2}$, $\boldsymbol{E_3}$ toward each of them
- we have placed the tails of the vectors at the point where the fields are to be evaluated

$$E_1 + E_2 = \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} + \frac{1}{4\pi\varepsilon_0}\frac{2Q}{d^2} = \frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2},$$

- The y components of two vectors cancel and the x components add → the net $\boldsymbol{E}$ at the origin is in the +ve x axis and has the magnitude

$$E = 2E_{3x} = 2E_3 \cos 30° = (2)\frac{1}{4\pi\varepsilon_0}\frac{4Q}{d^2}(0.866) = \frac{6.93Q}{4\pi\varepsilon_0 d^2}.$$



Questions:

2. Calculate the electric field at a distance 5cm from a point charge of 2.5 nC

$$E = \frac{kq}{r^2} = \frac{9 \times 10^9 \times 2.5 \times 10^{-9}}{0.05^2} = 9000 N/C$$

3. A point charge at 2cm produces an electric field of 180N/C. Calculate the magnitude of the electric field at 4 cm.

$$q = \frac{Er^2}{k} = \frac{180 \times 0.02^2}{9 \times 10^9} = 8 \times 10^{-12} C$$

$$E = \frac{kq}{r^2} = \frac{9 \times 10^9 \times 8 \times 10^{-12}}{0.04^2} = 45 N/C$$

Questions:

4. (a) Calculate the magnitude of the electric field at the point A.

$$E_+ = \frac{kq}{r^2} = \frac{9\times10^9 \times 3\times10^{-6}}{2^2} = 6750 N/C$$

$$E_- = \frac{kq}{r^2} = \frac{9\times10^9 \times 4\times10^{-6}}{5^2} = 1440 N/C$$

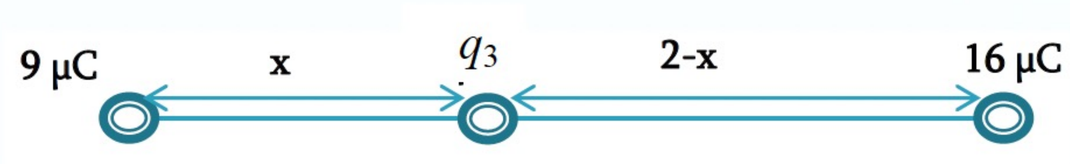
$$E = E_+ - E_- = 6750 - 1440 = 5310 N/C$$

(b) If a third charge 2μC is placed at A, calculate the force on it.

$$F = qE = 2\times10^{-6} \times 5310 = 0.01062 N$$

Questions:

5. Two charges 9 and 16μC are separated by a distance of 2m. Where should a third charge q_3 be placed between them for a net electric field on q_3 to be zero?



$$E = kq/r^2$$

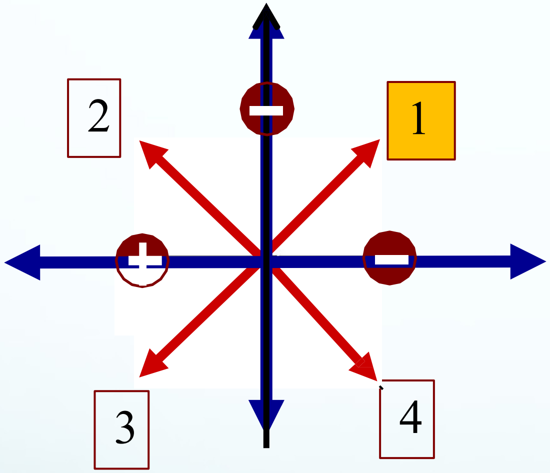
$$E_1 = E_2$$

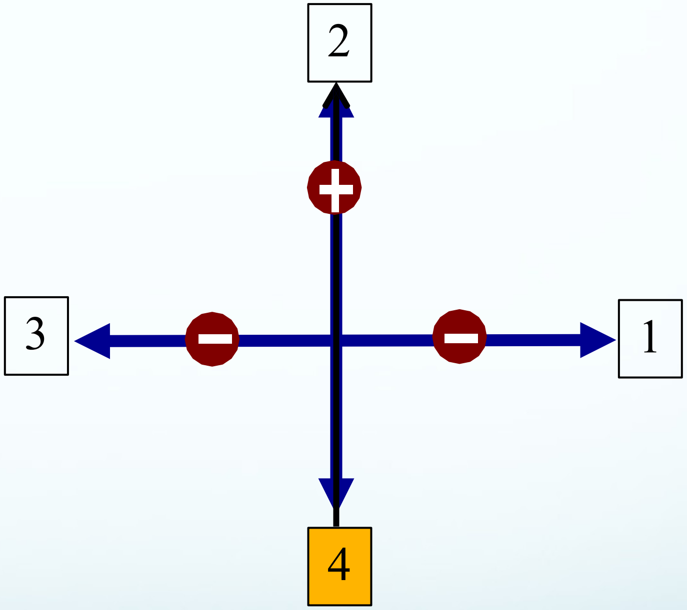
$$\frac{9\times10^9\times9\times10^{-6}}{x^2} = \frac{9\times10^9\times16\times10^{-6}}{(2-x)^2}$$

$$\frac{9}{x^2} = \frac{16}{(2-x)^2} \Rightarrow \frac{3}{x} = \frac{4}{(2-x)} \Rightarrow x = 0.86m$$

Questions:

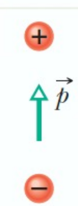
6. Three charges (one +ve and two are -ve) placed on the *x* and *y* axes as shown. What is the approximate direction of the electric field at the origin? Will it be pointing toward point 1, 2, 3, or 4?





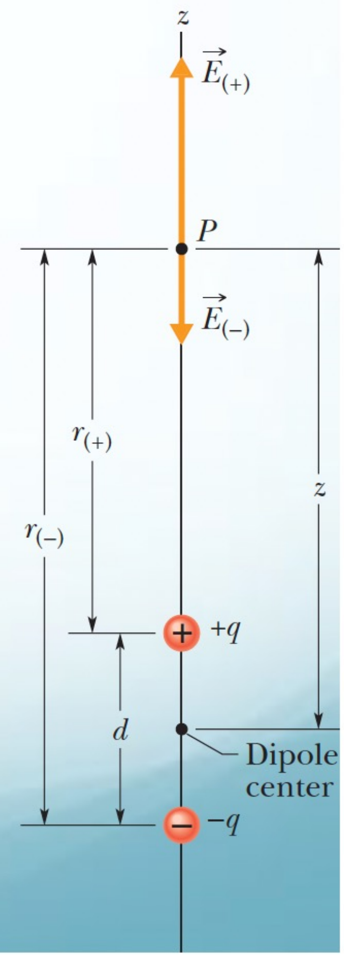
The Electric Field Due to an Electric Dipole

- ***Electric dipole:*** two charged particles of the same magnitude q but opposite sign, separated by a distance d



- ***Dipole axis:*** the axis through the particles

- ***Electric dipole moment*** $\vec{p}$***:*** vector quantity, its magnitude $p = qd$, its direction from the –ve to +ve end of the dipole



$$E = E_{(+)} - E_{(-)}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(+)}^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{r_{(-)}^2}$$

$$= \frac{q}{4\pi\varepsilon_0(z - \frac{1}{2}d)^2} - \frac{q}{4\pi\varepsilon_0(z + \frac{1}{2}d)^2} = \frac{q}{4\pi\varepsilon_0 z^2}\left(\frac{1}{\left(1 - \frac{d}{2z}\right)^2} - \frac{1}{\left(1 + \frac{d}{2z}\right)^2}\right)$$

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3}.$$

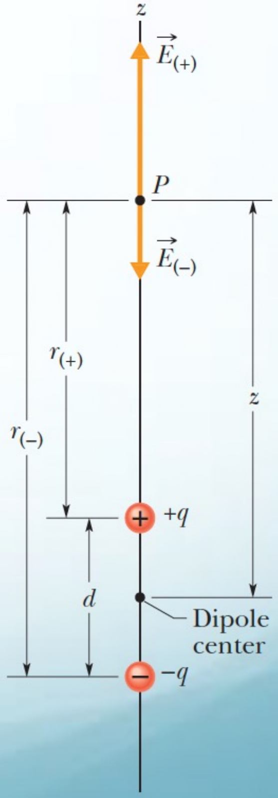
$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3} \qquad \text{(electric dipole).}$$

- $\boldsymbol{E}$ direction (for distant points on the dipole axis) = $\vec{\boldsymbol{p}}$ direction

Questions:

8. Two equal and opposite charges 6μC and -6μC are separated by a distance of 2cm. What is the magnitude of the electric field at 30cm from their midpoint?

$$E = \frac{1}{2\pi\varepsilon_0}\frac{qd}{z^3} = \frac{1}{2\times 3.14\times 8.85\times 10^{-12}}\frac{6\times 10^{-6}\times 0.02}{0.3^3} = 8\times 10^4\, N/C$$



The Electric Field Due to a Line of Charge

Continuous distributions of charges ≠ discrete distributions of charges : many closely spaced point charges that are spread along a line, over a surface, or within a volume

- We express the continuous distribution of charges on an object as a *charge density*

Table 22-2

Some Measures of Electric Charge

Name	Symbol	SI Unit
Charge	q	C
Linear charge density	λ	C/m
Surface charge density	σ	C/m^2
Volume charge density	ρ	C/m^3

How to find E due to line of charge (ring) at point *P*?

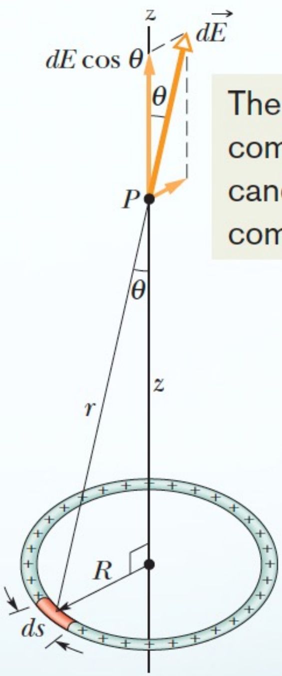
1. Divide the ring into differential elements of charge dq
2. Find dq: $dq = \lambda\, ds = \lambda 2\pi\, dR$
3. Find dE: $dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\lambda\, ds}{(z^2 + R^2)}$

$$dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\, ds.$$

4. Integrate dE $\quad E = \int dE\cos\theta = \frac{z\lambda}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}\int_0^{2\pi R} ds$

$$E = \frac{z\lambda(2\pi R)}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}$$

$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}} \quad \text{(charged ring)}$$



The perpendicular components just cancel but the parallel components add.

$r = \sqrt{z^2 + R^2}$

$\cos\theta = z/r$

- At large distance from center z >> R

→ $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}$ (charged ring at large distance).

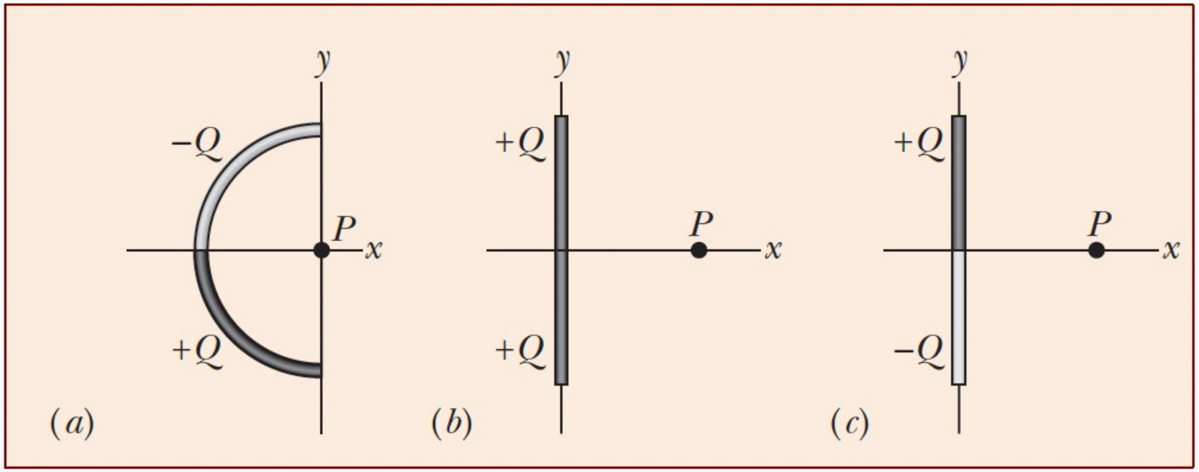
→ Charged ring at large distance looks like a point charge

- At the center of the ring $z = 0$ → $E = 0$

→ At the center of conductor there is no net force → no ***E***

CHECKPOINT 2

The figure here shows three nonconducting rods, one circular and two straight. Each has a uniform charge of magnitude Q along its top half and another along its bottom half. For each rod, what is the direction of the net electric field at point P?



(a) toward positive y;
(b) toward positive x;
(c) toward negative y

Questions:

9. A ring of radius 5cm carries a linear charge density of 5 × 10⁻⁶ C/m. Calculate the electric field at (a) 6cm, and (b) 6m from its center

(a)

$$E = \frac{1}{4\pi\varepsilon_0}\frac{qz}{\left(z^2+R^2\right)^{3/2}}$$

$$q = \lambda L = \lambda 2\pi R$$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{(\lambda 2\pi R)z}{\left(z^2+R^2\right)^{3/2}} = \frac{\lambda R z}{2\varepsilon_0\left(z^2+R^2\right)^{3/2}} = 1.8\times 10^6\, N/C$$

(b)

At 6m, z >>R

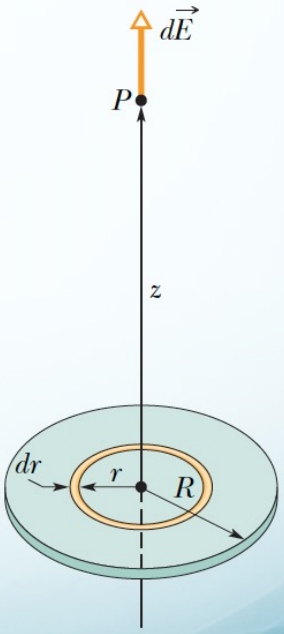
$$E = \frac{q}{4\pi\varepsilon_0 z^2} = \frac{\lambda R}{2\varepsilon_0 z^2} = 392 N/C$$

The Electric Field Due to a Charged Disk

- The disk is divided into flat rings with radius *r* and radial width *dr*

1. Find *dq*: $dq = \sigma\, dA = \sigma(2\pi r\, dr)$,
2. Find *dE*: $dE = \dfrac{z\sigma 2\pi r\, dr}{4\pi\varepsilon_0(z^2 + r^2)^{3/2}} = \dfrac{\sigma z}{4\varepsilon_0}\dfrac{2r\, dr}{(z^2 + r^2)^{3/2}}$, $E = \dfrac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}$ (charged ring)
3. Integrate *dE* $E = \int dE = \dfrac{\sigma z}{4\varepsilon_0}\int_0^R (z^2 + r^2)^{-3/2}(2r)\, dr.$

$$E = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right) \quad \text{(charged disk)}$$



- *E* at large distance from center z >> R, *E* = 0
- *E* at the center *z* =0 → $E = \dfrac{\sigma}{2\varepsilon_0}$
- If R → ∞, the second term in the parentheses→ zero

$$\rightarrow \quad E = \frac{\sigma}{2\varepsilon_0} \quad \text{(infinite sheet).}$$

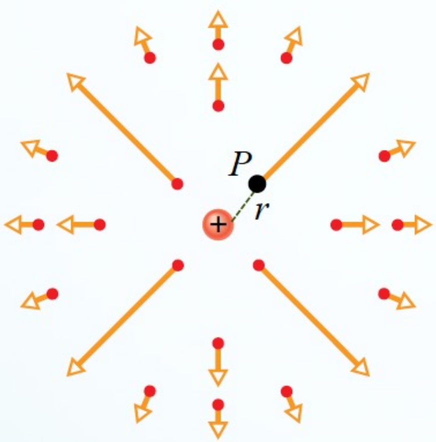
Questions:

10. The magnitude of the electric field at 5cm from an infinite sheet is 200N/C, what is its surface charge density?

$$E = \frac{\sigma}{2\varepsilon_0} \Rightarrow \sigma = 2\varepsilon_0 E = 2 \times 8.85 \times 10^{-12} \times 200 = 3.54 \ \text{nC/m}^2$$

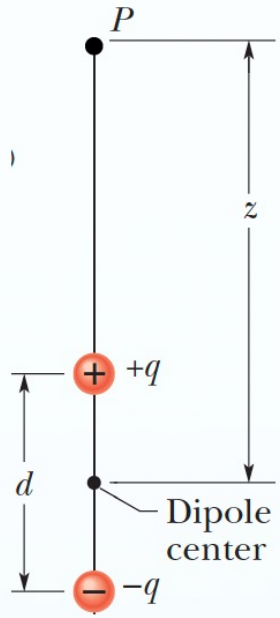
Electric Field at point *P* Due to:

Point Charge



$$E = \frac{1}{4\pi\varepsilon_o}\frac{|q|}{r^2}$$

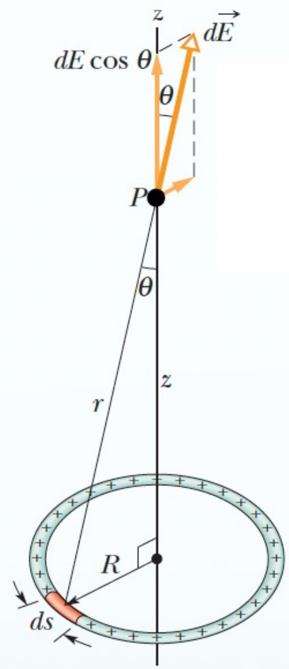
Electric Dipole



$$E = \frac{1}{2\pi\varepsilon_o}\frac{qd}{z^3}$$

Line of Charge

$dq = \lambda\, ds = \lambda 2\pi\, dR$



$$E = \frac{z\lambda(2\pi R)}{4\pi\varepsilon_o(z^2+R^2)^{3/2}}$$

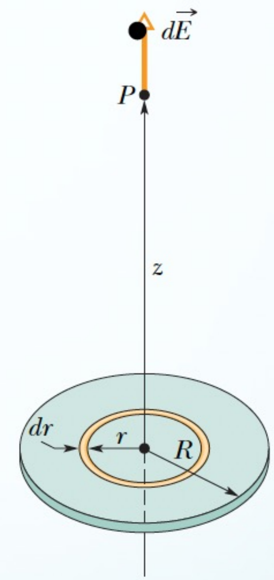
$$E = \frac{qz}{4\pi\varepsilon_0(z^2 + R^2)^{3/2}}$$

At z >> R → $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{z^2}$

At $z = 0$ → $E = 0$

Charged Disk

$dq = \sigma\, dA = \sigma(2\pi r\, dr),$



$$E = \frac{\sigma}{2\varepsilon_o}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right)$$

At z >> R → $E = 0$

At $z = 0$ → $E = \frac{\sigma}{2\varepsilon_0}$

If R → ∞ → $E = \frac{\sigma}{2\varepsilon_0}$ (infinite sheet)

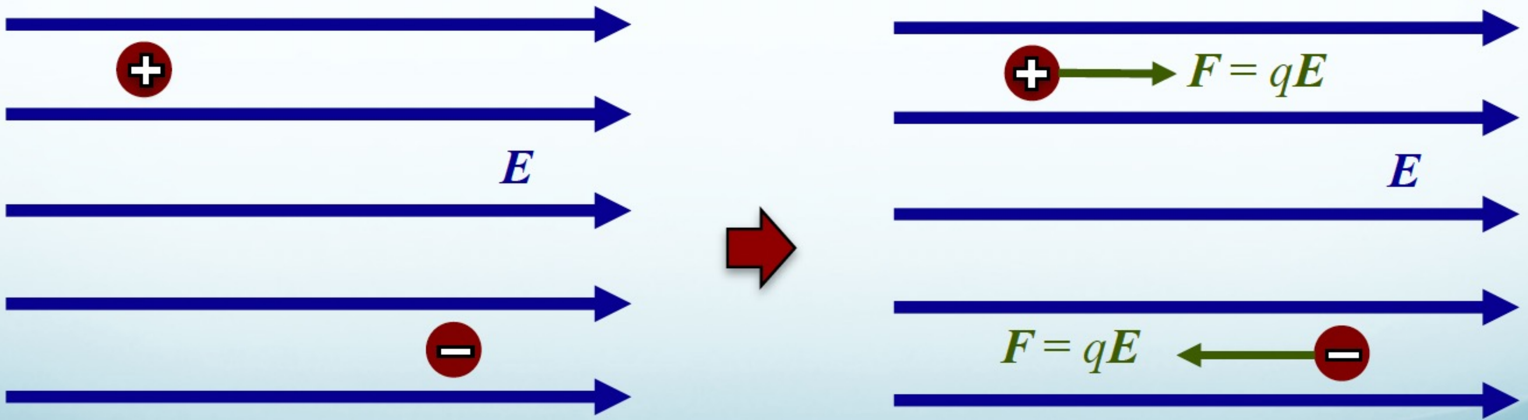
A Point Charge in an Electric Field

What happens if a charged particle is placed in an external electric field?

→ Electrostatic force will act on the particle: $\vec{F} = q\vec{E},$

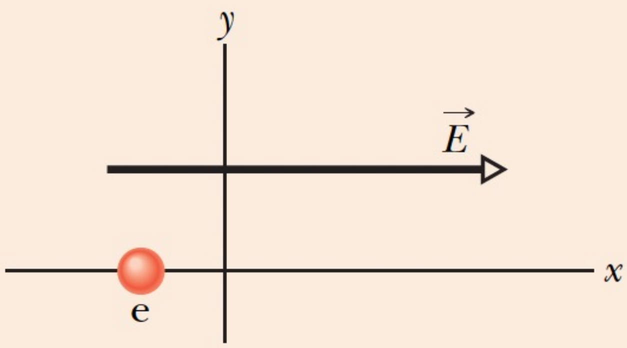
q: the charge of the particle
E: electric field from another charges, it is called *external field*

The electrostatic force $\vec{F}$ acting on a charged particle located in an external electric field $\vec{E}$ has the direction of $\vec{E}$ if the charge q of the particle is positive and has the opposite direction if q is negative.



CHECKPOINT 3

(a) In the figure, what is the direction of the electrostatic force on the electron due to the external electric field shown? (b) In which direction will the electron accelerate if it is moving parallel to the *y* axis before it encounters the external field? (c) If, instead, the electron is initially moving rightward, will its speed increase, decrease, or remain constant?



a) Leftward
b) Leftward
c) Decrease