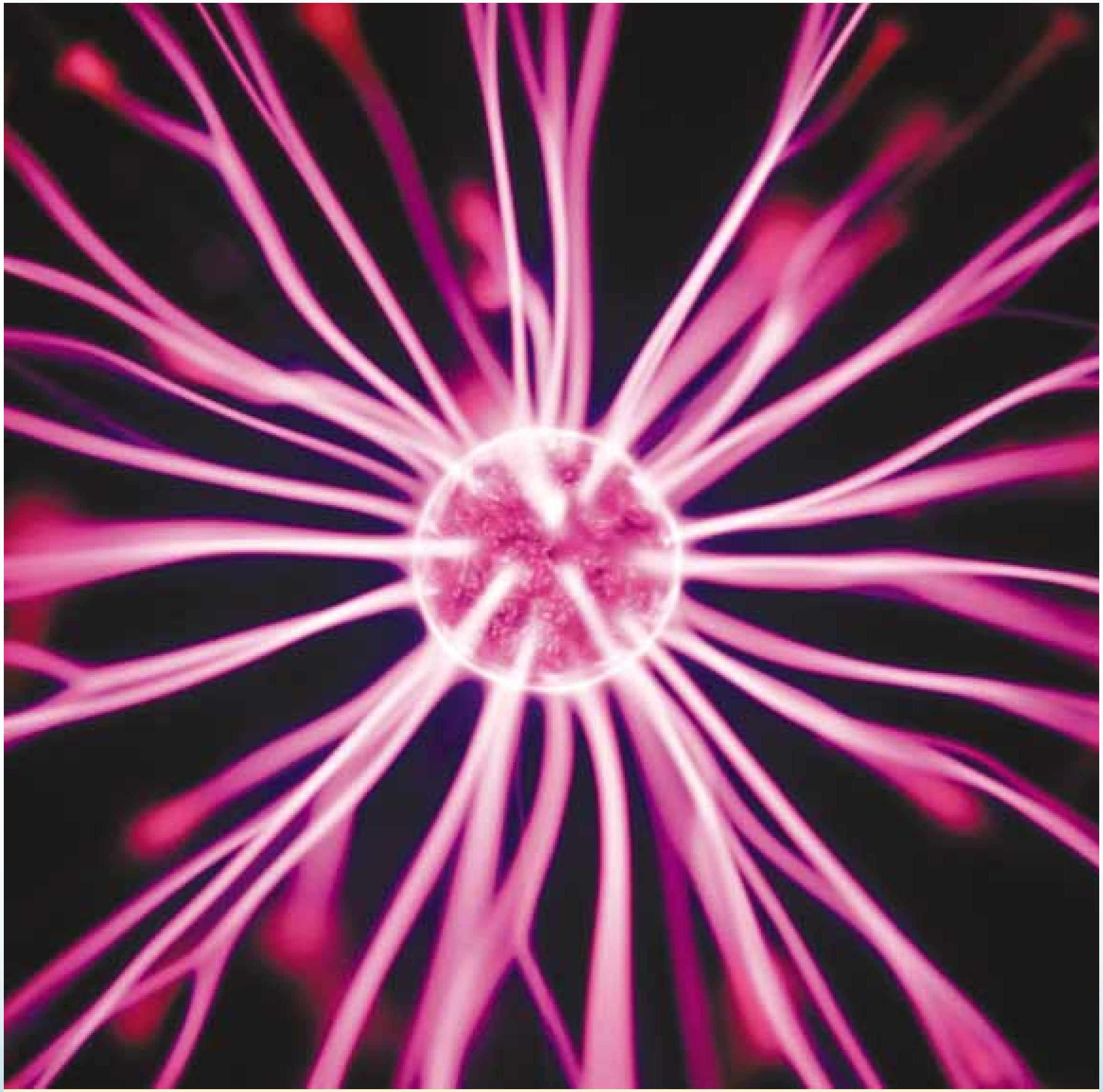
General Physics 2
SCPH 211

Chapter 23
GAUSS' LAW



▲ *In a table-top plasma ball, the colorful lines emanating from the sphere give evidence of strong electric fields. Using Gauss's law, we show in this chapter that the electric field surrounding a charged sphere is identical to that of a point charge. (Getty Images)*

Outline:

- Introduction
- Flux
- Flux of an Electric Field
- Gauss' Law
- Gauss' Law and Coulomb's Law
- A Charged Isolated Conductor
- Applying Gauss' Law

To find E due to different charged objects (from chapter 22)

1. Find dq
2. Find dE
3. Integrate dE

- For symmetrical charge distributions → **Gauss' law** is used to find E

- ***To find E using Gauss' law ?***
  1. Choose a Gaussian surface
  2. Find electric flux Φ
  3. Find enclosed charge q_{enc}
  4. Calculate E

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(electric flux through a Gaussian surface).}$$

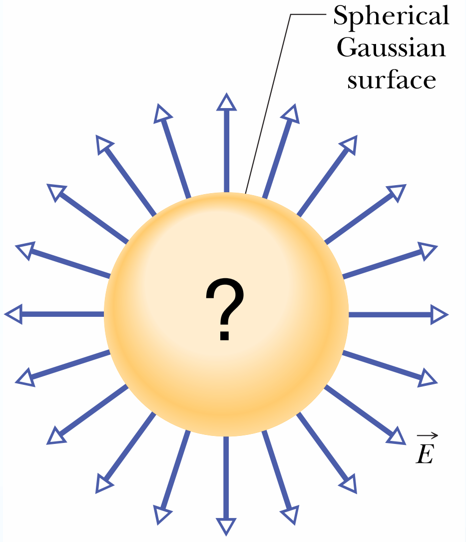
$$\varepsilon_0 \Phi = q_{enc} \qquad \text{(Gauss' law).}$$

- ***Gaussian surface:***
  - A [1]·hypothetical (imaginary) [2]·closed surface [3]·enclosing the charges
  - Can have [4]·any shape (better to be symmetry)
  - e.g., if the charge is spread uniformly over a sphere
    → spherical Gaussian surface

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(electric flux through a Gaussian surface).}$$

Gauss' law relates the electric fields at points on a (closed) Gaussian surface to the net charge enclosed by that surface.

$$\varepsilon_0 \Phi = q_{enc} \qquad \text{(Gauss' law).}$$

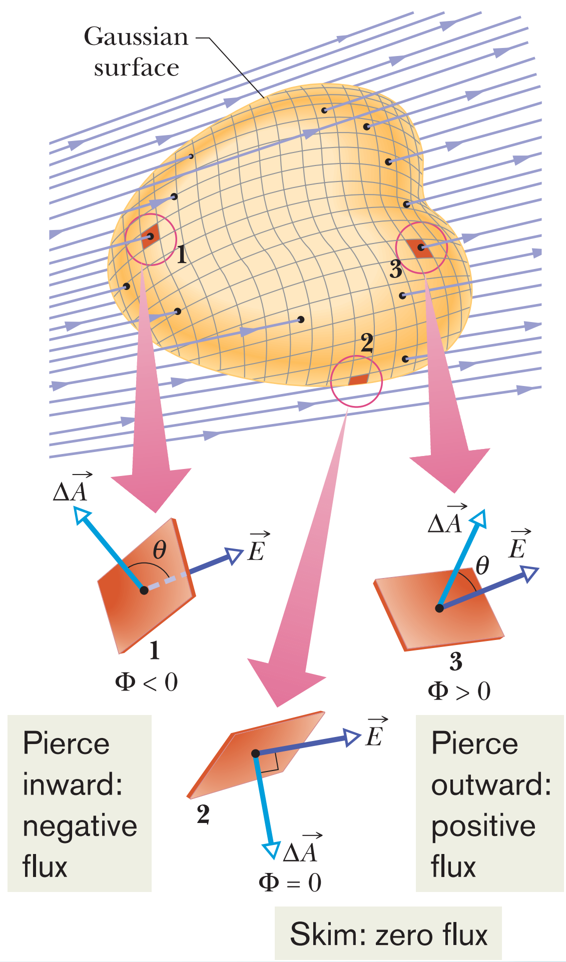


- If we know the electric field on a Gaussian surface → we can evaluate q_{enc}
- e.g., Fig. shows the electric field vectors → Gauss' law tells us that the spherical surface must enclose net +ve charge
- However, to calculate how much charge is enclosed, we need to calculate how much electric field is intercepted by the Gaussian surface
- This measure of intercepted field is called ***flux***

Flux of an Electric Field

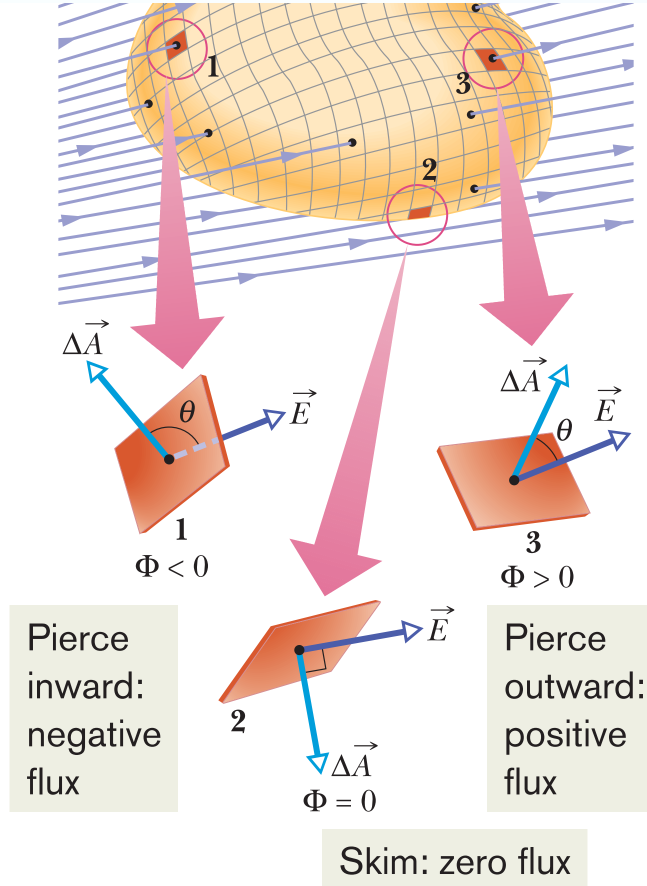
- An arbitrary (asymmetric) Gaussian surface immersed in a nonuniform electric field
- The surface is divided into small squares of flat area ΔA
- Each element of area has an area vector $\Delta \boldsymbol{A}$
  - *Magnitude*: the area ΔA
  - *Direction*: perpendicular & away to Gaussian surface
- E is constant for any given square area
- The vectors $\Delta \boldsymbol{A}$ & $\boldsymbol{E}$ make angle θ with each other
- The flux of the electric field for the Gaussian surface is

$$\Phi = \sum \vec{E} \cdot \Delta \vec{A}.$$



- The value of each scalar product (+ve, -ve, or 0) → Φ is +ve, -ve, or 0
  - For square 1 , vector *E* points inward → -ve contribution
  - For square 2, vector *E* lies in the surface → zero contribution
  - For squares 3, vector *E* points outward → +ve contribution

$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(electric flux through a Gaussian surface).}$$



- The flux of the electric field is a *scalar quantity*
  The flux SI unit is (N m^2/C)

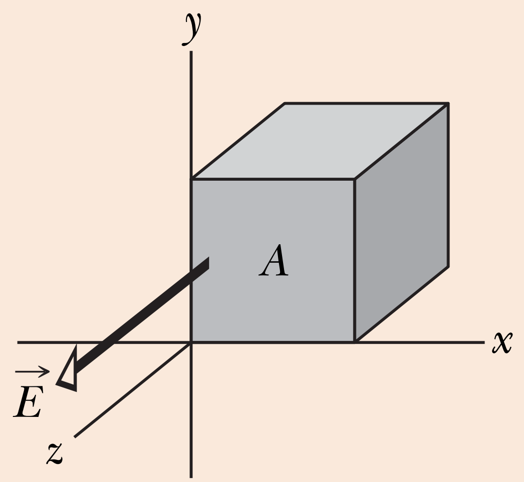
The electric flux Φ through a Gaussian surface is proportional to the net number of electric field lines passing through that surface.

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

- (a) +EA
- (b) -EA
- (c) 0
- (d) 0

CHECKPOINT 1

The figure here shows a Gaussian cube of face area A immersed in a uniform electric field $\vec{E}$ that has the positive direction of the z axis. In terms of E and A, what is the flux through (a) the front face (which is in the xy plane), (b) the rear face, (c) the top face, and (d) the whole cube?

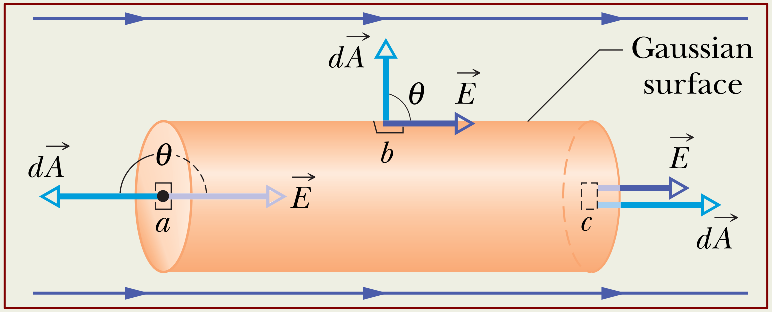


Sample Problem

Figure 23-4 shows a Gaussian surface in the form of a cylinder of radius R immersed in a uniform electric field $\vec{E}$, with the cylinder axis parallel to the field. What is the flux Φ of the electric field through this closed surface?

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_a \vec{E} \cdot d\vec{A} + \int_b \vec{E} \cdot d\vec{A} + \int_c \vec{E} \cdot d\vec{A}. \qquad (23\text{-}5)$$



For all points on the left cap, the angle θ between $\vec{E}$ and $d\vec{A}$ is 180° and the magnitude E of the field is uniform. Thus,

$$\int_a \vec{E} \cdot d\vec{A} = \int E(\cos 180°)\, dA = -E \int dA = -EA,$$

where $\int dA$ gives the cap's area A $(= \pi R^2)$. Similarly, for the right cap, where $\theta = 0$ for all points,

$$\int_c \vec{E} \cdot d\vec{A} = \int E(\cos 0)\, dA = EA.$$

Finally, for the cylindrical surface, where the angle θ is 90° at all points,

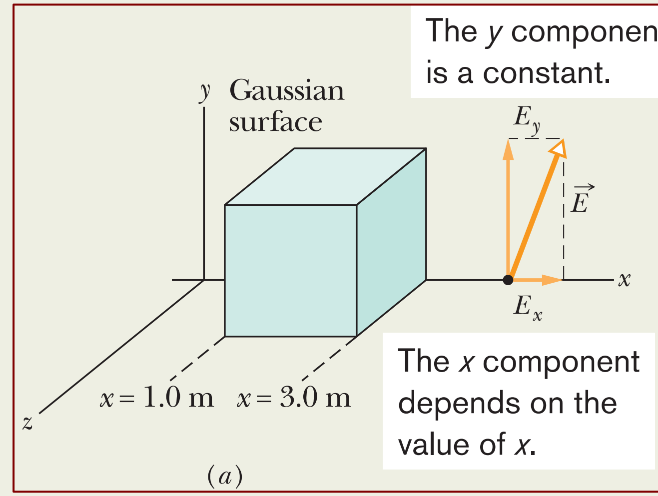
$$\int_b \vec{E} \cdot d\vec{A} = \int E(\cos 90°)\, dA = 0.$$

Substituting these results into Eq. 23-5 leads us to

$$\Phi = -EA + 0 + EA = 0. \qquad \text{(Answer)}$$

Sample Problem

A *nonuniform* electric field given by $\vec{E} = 3.0x\hat{i} + 4.0\hat{j}$ pierces the Gaussian cube shown in Fig. 23-5*a*. (*E* is in newtons per coulomb and *x* is in meters.) What is the electric flux through the right face, the left face, and the top face? (We consider the other faces in another sample problem.)



Right face:

$$d\vec{A} = dA\hat{i}.$$

From Eq. 23-4, the flux Φ_r through the right face is then

$$\Phi_r = \int \vec{E} \cdot d\vec{A} = \int (3.0x\hat{i} + 4.0\hat{j}) \cdot (dA\hat{i})$$

$$= \int [(3.0x)(dA)\hat{i} \cdot \hat{i} + (4.0)(dA)\hat{j} \cdot \hat{i}]$$

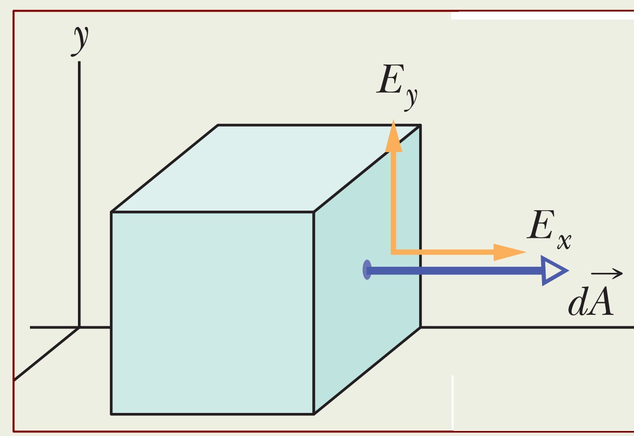
$$= \int (3.0x\, dA + 0) = 3.0 \int x\, dA.$$

$x = 3.0$ m.

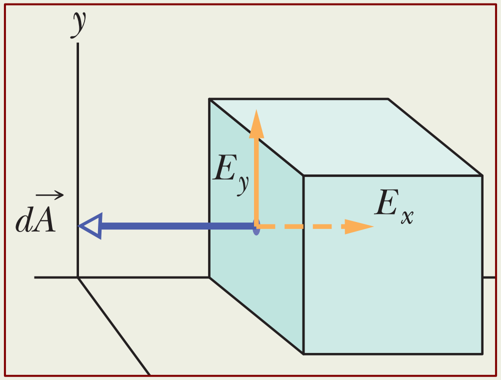
Square area = 2^2

$$\Phi_r = 3.0 \int (3.0)\, dA = 9.0 \int dA$$

$$\Phi_r = (9.0\text{ N/C})(4.0\text{ m}^2) = 36\text{ N} \cdot \text{m}^2/\text{C}.$$



Left face:



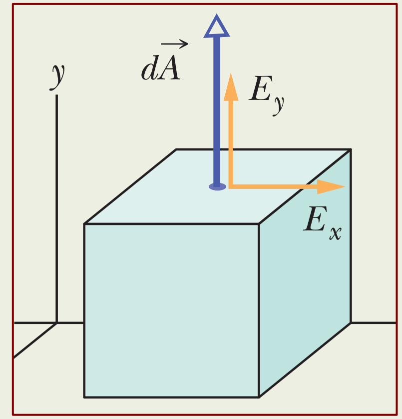
$$d\vec{A} = -dA\hat{\mathrm{i}}$$

$$x = 1.0 \text{ m.}$$

$$\Phi_l = -3.0 \int x\, dA.$$

$$\Phi_l = -12 \text{ N} \cdot \text{m}^2/\text{C}.$$

Top face:



$$d\vec{A} = dA\hat{\mathrm{j}}$$

$$\Phi_t = \int (3.0x\hat{\mathrm{i}} + 4.0\hat{\mathrm{j}}) \cdot (dA\hat{\mathrm{j}})$$

$$= \int [(3.0x)(dA)\hat{\mathrm{i}} \cdot \hat{\mathrm{j}} + (4.0)(dA)\hat{\mathrm{j}} \cdot \hat{\mathrm{j}}]$$

$$= \int (0 + 4.0\, dA) = 4.0 \int dA$$

$$= 16 \text{ N} \cdot \text{m}^2/\text{C}.$$

Gauss' Law

Gauss' law relates the net flux Φ of an electric field through a Gaussian surface to the *net* charge q_{enc} that is *enclosed* by that surface

$$\varepsilon_0 \Phi = q_{enc} \qquad \text{(Gauss' law).}$$

$\varepsilon_o = 8.85 \times 10^{-12}\ C^2/N.m^2$

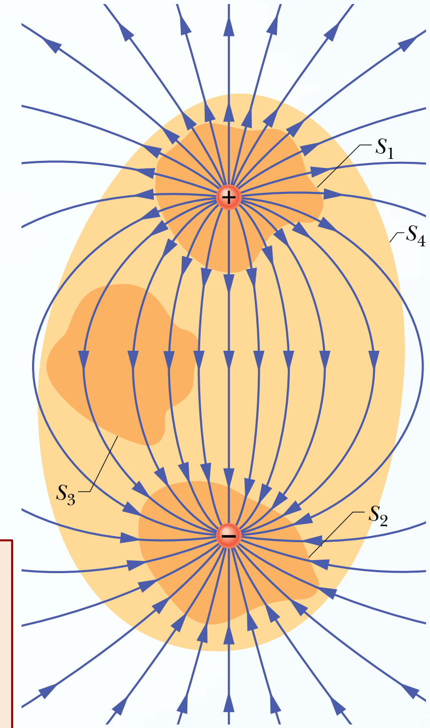
$$\Phi = \oint \vec{E} \cdot d\vec{A} \qquad \text{(electric flux through a Gaussian surface).}$$

→

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \qquad \text{(Gauss' law).}$$

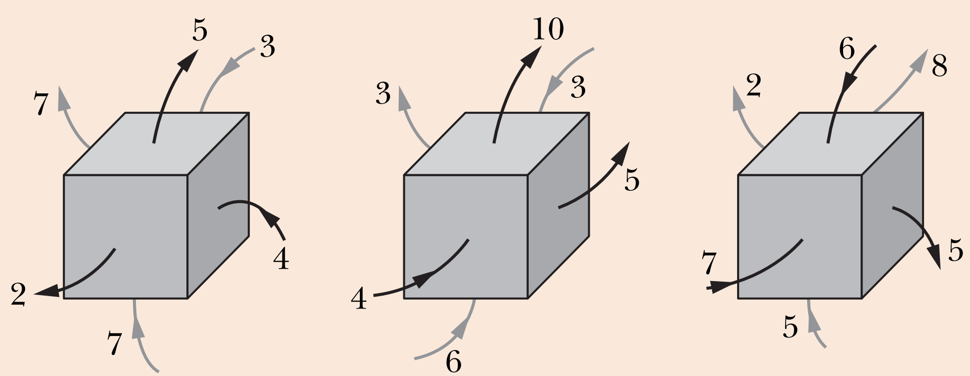
- *E*: resulting from all charges inside & outside the Gaussian surface
- net q_{enc} :
  - Algebraic sum of all the enclosed +ve & -ve charges
  - Can be +ve, -ve, or zero
  - Its sign is important:
    - If q_{enc} is +ve → *E* is outward
    - If q_{enc} is -ve → *E* is inward
  - Charge outside the surface is not included in q_{enc}
  - The exact form & location of the charges inside the Gaussian surface are not important

- Surface S_1: E outward → + Φ → + q_{enc}
- Surface S_2: E inward → - Φ → - q_{enc}
- Surface S_3: no charge → $q_{enc} = 0$ → $\Phi = 0$
- Surface S_4: no net charge because $q_{enc} = -q + q = 0$ → $\Phi = 0$



CHECKPOINT 2

The figure shows three situations in which a Gaussian cube sits in an electric field. The arrows and the values indicate the directions of the field lines and the magnitudes (in $N \cdot m^2/C$) of the flux through the six sides of each cube. (The lighter arrows are for the hidden faces.) In which situation does the cube enclose (a) a positive net charge, (b) a negative net charge, and (c) zero net charge?

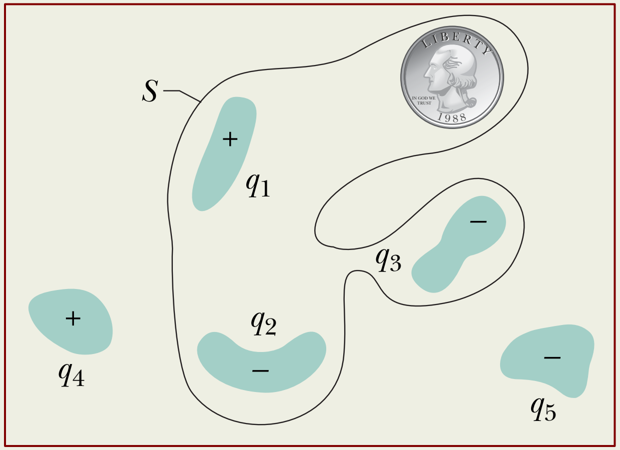


(1) (2) (3)

0 +ve -ve

Sample Problem

Figure 23-7 shows five charged lumps of plastic and an electrically neutral coin. The cross section of a Gaussian surface S is indicated. What is the net electric flux through the surface if $q_1 = q_4 = +3.1$ nC, $q_2 = q_5 = -5.9$ nC, and $q_3 = -3.1$ nC?



$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \frac{q_1 + q_2 + q_3}{\varepsilon_0}$$

$$= \frac{+3.1 \times 10^{-9}\,\text{C} - 5.9 \times 10^{-9}\,\text{C} - 3.1 \times 10^{-9}\,\text{C}}{8.85 \times 10^{-12}\,\text{C}^2/\text{N}\cdot\text{m}^2}$$

$$= -670\,\text{N}\cdot\text{m}^2/\text{C}. \qquad \text{(Answer)}$$

Sample Problem

What is the net charge enclosed by the Gaussian cube of Fig. 23-5, which lies in the electric field $\vec{E} = 3.0x\hat{\text{i}} + 4.0\hat{\text{j}}$? ($E$ is in newtons per coulomb and x is in meters.)

$$\varepsilon_0\Phi = q_{\text{enc}}$$

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

Flux:

right face ($\Phi_r = 36\ \text{N}\cdot\text{m}^2/\text{C}$), the left face ($\Phi_l = -12\ \text{N}\cdot\text{m}^2/\text{C}$), and the top face ($\Phi_t = 16\ \text{N}\cdot\text{m}^2/\text{C}$).

bottom face,

$d\vec{A} = -dA\hat{\text{j}}$, and we find

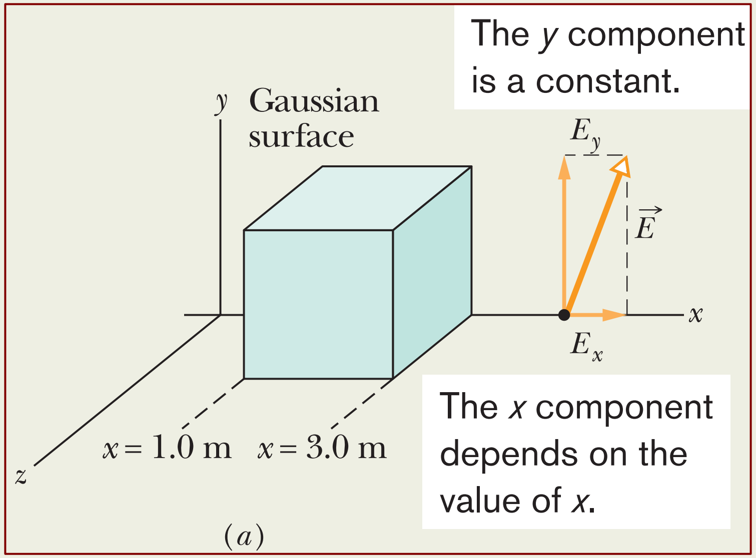
$$\Phi_b = -16\ \text{N}\cdot\text{m}^2/\text{C}.$$

For the front face we have $d\vec{A} = dA\hat{\text{k}}$, and for the back face, $d\vec{A} = -dA\hat{\text{k}}$. When we take the dot product of the given electric field $\vec{E} = 3.0x\hat{\text{i}} + 4.0\hat{\text{j}}$ with either of these expressions for $d\vec{A}$, we get 0 and thus there is no flux through those faces. We can now find the total flux through the six sides of the cube:

$$\Phi = (36 - 12 + 16 - 16 + 0 + 0)\ \text{N}\cdot\text{m}^2/\text{C}$$
$$= 24\ \text{N}\cdot\text{m}^2/\text{C}.$$

Enclosed charge: Next, we use Gauss' law to find the charge q_{enc} enclosed by the cube:

$$q_{\text{enc}} = \varepsilon_0\Phi = (8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(24\ \text{N}\cdot\text{m}^2/\text{C})$$
$$= 2.1 \times 10^{-10}\ \text{C}. \qquad \text{(Answer)}$$

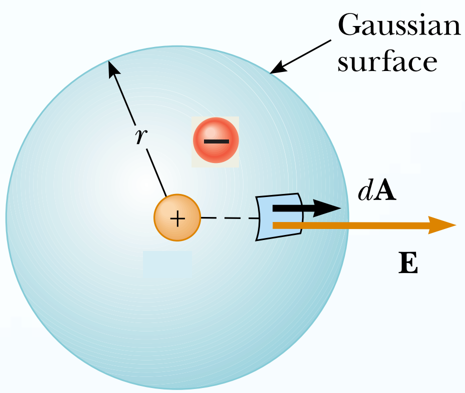


(a)

Examples:

Q.1. Two charges 25.9μC and -8.2 μC are confined in a spherical surface of radius *r* = 5cm. Calculate the net electric flux through the surface. Calculate the magnitude of the electric field at the surface.

$$\Phi = \frac{q_{enc}}{\varepsilon_o} = \frac{(25.9-8.2)\times10^{-6}}{8.85\times10^{-12}} = 2\times10^{6} \text{N.m}^2/\text{C}$$



$$\Phi = \vec{E}.\vec{A} = EAcos0 = EA$$

$$\rightarrow E = \Phi/A = \frac{2\times10^{6}}{4(3.14)(0.05)^2} = 6.4\times10^{7} \text{N/C}$$

Q.2. A certain charge is enclosed in a sphere of radius R. If the electric flux through the sphere is 450 N.m^2/C, calculate the charge *q*.

$$\Phi = \frac{q_{enc}}{\varepsilon_o} = \rightarrow q = \Phi\varepsilon_o = 450\times8.85\times10^{-12} = 3.98nC$$

Examples:

Q.3. An 8m² plate is immersed in a uniform electric field of 2000N/C. If the plane of the plate makes an angle of 90° with the electric field, calculate the electric flux and find the enclosed charge.

Answer:

Since the plane makes an angle 90° with the electric field, this means the area unit vector $\vec{A}$ makes Zero angle with the electric field

$$\Phi = \vec{E}.\vec{A} = EAcos0 = 16\times10^{3}N.\frac{m^2}{C}$$

$$\Phi = \frac{q_{enc}}{\varepsilon_o} \rightarrow q_{enc} = \Phi\varepsilon_o = 16\times10^{3}\times8.85\times10^{-12} = 141.6nC$$

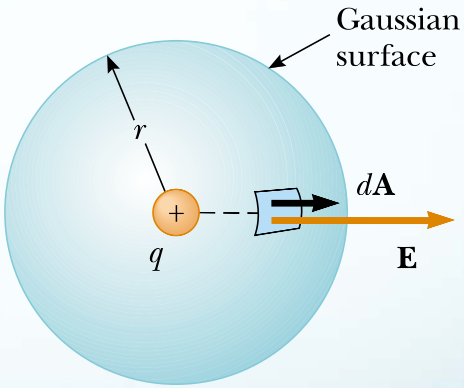
Gauss' Law and Coulomb's Law

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc} \quad \text{(Gauss' law).}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\,\hat{\mathrm{r}} \quad \text{(point charge).}$$

- Gauss' law & Coulomb's law relate q & E in <u>static</u> situations

- Deriving Coulomb's law from Gauss' law:
  - A spherical Gaussian surface of radius r is drawn around $+q$
  - Gaussian surface is divided into differential areas dA
  - Vector dA is perpendicular to the surface & directed outward
  - $\boldsymbol{E}$ is also perpendicular to the surface and directed outward → θ between $\boldsymbol{E}$ & d$\boldsymbol{A}$ = zero



$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = \varepsilon_0 \oint E\, dA = q_{enc}.$$

  - $q_{enc} = q$, E is constant → $\varepsilon_0 E \oint dA = q.$ → $\varepsilon_0 E(4\pi r^2) = q$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}.$$

CHECKPOINT 3

There is a certain net flux Φ_i through a Gaussian sphere of radius r enclosing an isolated charged particle. Suppose the enclosing Gaussian surface is changed to (a) a larger Gaussian sphere, (b) a Gaussian cube with edge length equal to r, and (c) a Gaussian cube with edge length equal to $2r$. In each case, is the net flux through the new Gaussian surface greater than, less than, or equal to Φ_i?

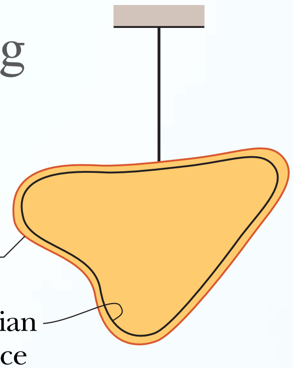
$$\varepsilon_0 \Phi = q_{enc}$$

(a) equal;
(b) equal;
(c) equal

A Charged Isolated Conductor

If an excess charge is placed on an isolated conductor, that amount of charge will move entirely to the surface of the conductor. None of the excess charge will be found within the body of the conductor.

Figure shows isolated, charged piece of copper hanging from an insulating thread & having an excess charge q



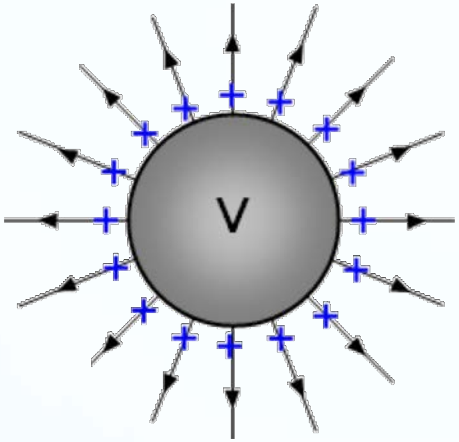
The Gaussian surface just inside the surface of the conductor

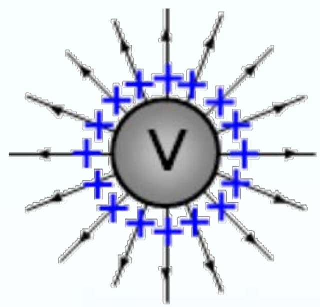
- E = zero everywhere <u>inside</u> a conductor
  - If this were not true, $\boldsymbol{E}$ would exert $\boldsymbol{F}$ on free electrons of the conductor → current would always exist within a conductor
- → E = zero for all points <u>on</u> the Gaussian surface
- → Φ through the Gaussian surface = zero

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{\text{enc}} \qquad \text{(Gauss' law).}$$

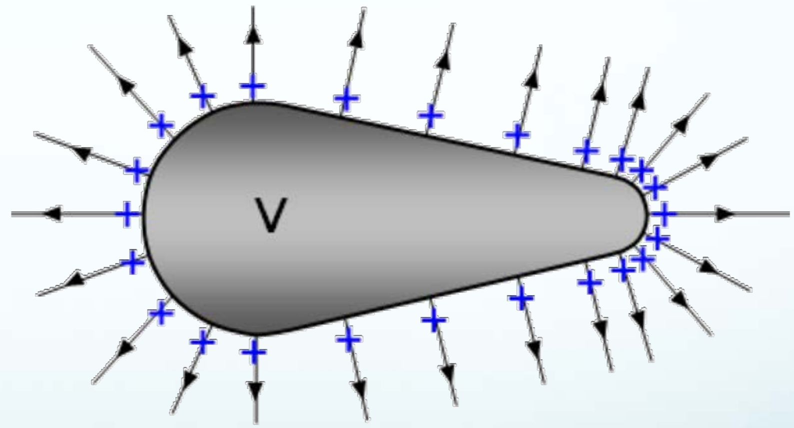
- → the net q_{enc} = zero
  → the excess charge must be outside the Gaussian surface
  → it must lie on the actual surface of the conductor

The External Electric Field





$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

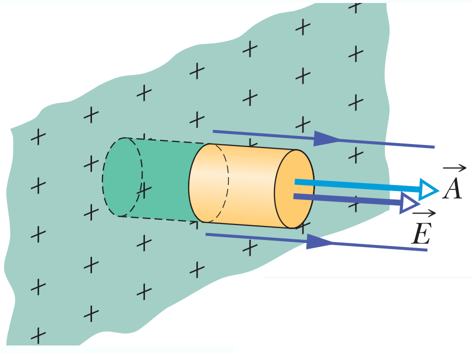


- If conductor is spherical
  → q distribute uniformly

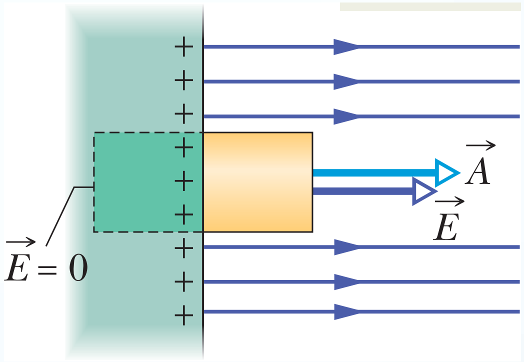
- If conductor is not spherical
  → q distribute not uniformly
  →σ (charge q/area A) varies everywhere over the surface of any nonspherical conductor
  → Difficult to find ***E*** that sets up by the surface charges

Applying Gauss' Law

Electric field due to a charged conductor



- Consider a section of small surface area A of conductor with excess +ve charge
- Gaussian surface is a cylinder perpendicular to the conductor's surface
- One end cap is fully inside the conductor, the other is fully outside
- E outside the conductor is perpendicular to its surface



$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{enc}}}{\varepsilon_0}$$

- Φ_1 (through the inner end of cylinder) = 0
- Φ_2 (through the curved surface of cylinder) = 0
- Φ_3 (through the outer end of cylinder) = $EA\cos 0 = EA = q_{enc}/\varepsilon_0$
- $\sigma = q_{enc}/A \rightarrow q_{en} = \sigma A$
- $\rightarrow EA = \sigma A/\varepsilon_0$
- $\rightarrow$

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{(conducting surface).}$$

Sample Problem

Figure 23-11*a* shows a cross section of a spherical metal shell of inner radius R. A point charge of $-5.0\ \mu C$ is located at a distance $R/2$ from the center of the shell. If the shell is electrically neutral, what are the (induced) charges on its inner and outer surfaces? Are those charges uniformly distributed? What is the field pattern inside and outside the shell?

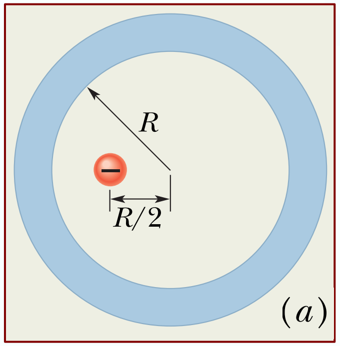
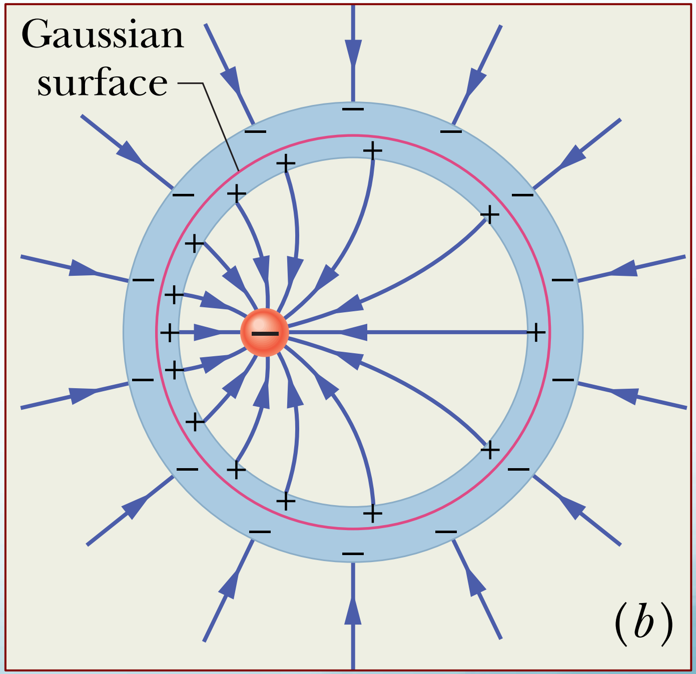


Figure 23-11*b* shows a cross section of a spherical Gaussian surface within the metal, just outside the inner wall of the shell. The electric field must be zero inside the metal (and thus on the Gaussian surface inside the metal). This means that the electric flux through the Gaussian surface must also be zero. Gauss' law then tells us that the *net* charge enclosed by the Gaussian surface must be zero.

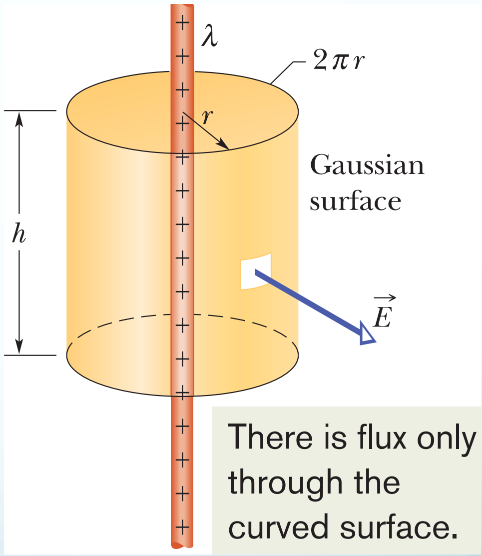


Electric field due to a charged plastic rod

An infinite long cylindrical plastic rod with a uniform +ve linear charge density λ ($\lambda = q_{enc}/h$)

- To find ***E*** at a distance *r* from the rod axis:
  - We chose Gaussian surface as cylinder of radius *r* & length *h*
  - At every point on the Gaussian surface, ***E*** directed radially outward
  - → Φ through the cylindrical surface is:

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \qquad (q_{enc} = \lambda h)$$



  - The area *A* of the cylindrical surface is $2\pi rh$

$$\varepsilon_0 E(2\pi rh) = \lambda h,$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r} \qquad \text{(line of charge).}$$

Examples:

Q.4 If the electric field at 10cm from a long-straight wire is 20N/C, what is the electric field at 2cm from the wire is:

$$E = \frac{\lambda}{2\pi\varepsilon_o r} \Longrightarrow \lambda = 2\pi\varepsilon_o r E$$

At 10cm = 0.1m, $\lambda = 2\pi\varepsilon_o(0.1)(20)$

At 2cm=0.02m, $\lambda = 2\pi\varepsilon_o(0.02)(E_2)$

$$\lambda = \lambda$$

$$\rightarrow 2\pi\varepsilon_o(0.1)(20) = 2\pi\varepsilon_o(0.02)(E_2)$$

$$\rightarrow E_2 = \frac{0.1 \times 20}{.02} = 100\text{N/C}$$

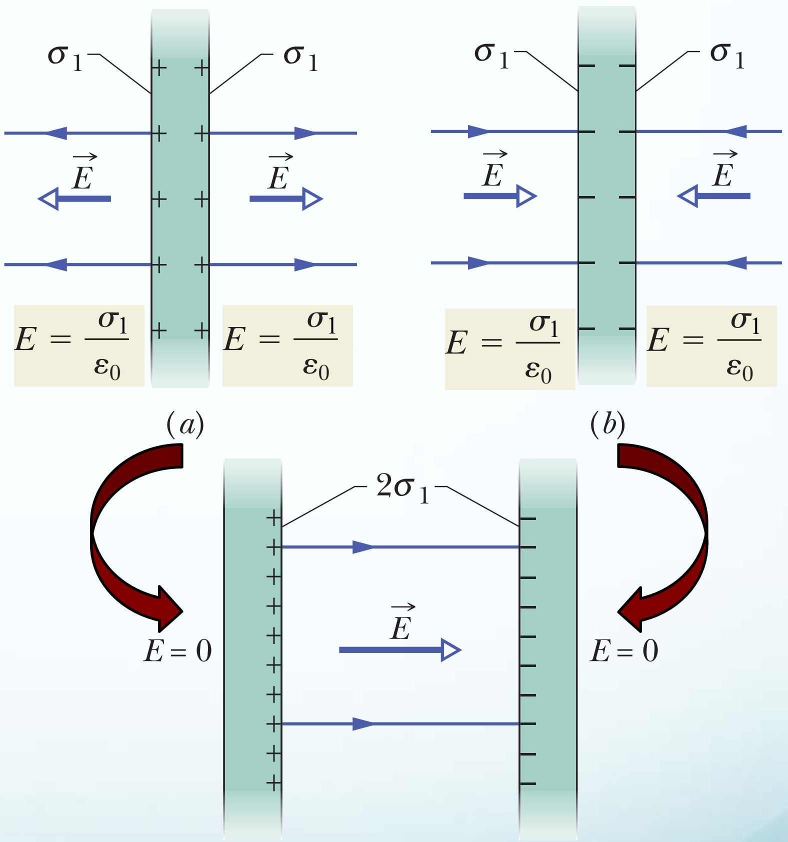
Electric field between two oppositely charged conducting plates

- For a conducting plate with charge density σ, charges will distribute uniformly on the plate surfaces
  → each surface of plate has $\sigma_1 = \frac{\sigma}{2}$

- If two conducting plates with same & opposite surface charges σ are placed parallel to each other
  - → all the excess charge moves onto the inner faces of the plates (due to attraction)
  - The new surface charge density σ on each inner face is twice σ_1
  - The magnitude of ***E*** between the plates is

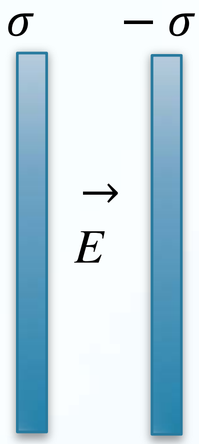
$$E = \frac{2\sigma_1}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

  - The direction of ***E*** from +ve plate to –ve one
  - Since no excess charge on the outer faces
    → $E = 0$ outside the plates



Examples:

Q.5 Two parallel conducting plates carry equal but opposite surface charges of 8.85 nC/m². Calculate the electric field between them.

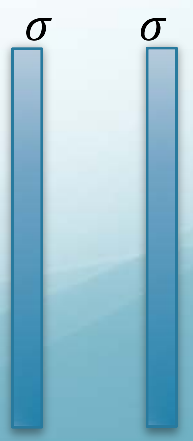


The electric field between the two oppositely charged conducting plate is

$$E = \frac{\sigma}{\varepsilon_0}$$

$$= \frac{8.85\times10^{-9}}{8.85\times10^{-12}} = 1000\mathrm{N/C}$$

Q.6 Two parallel conducting plates carry equal surface charges of 8.85 nC/m². Calculate the electric field between them.

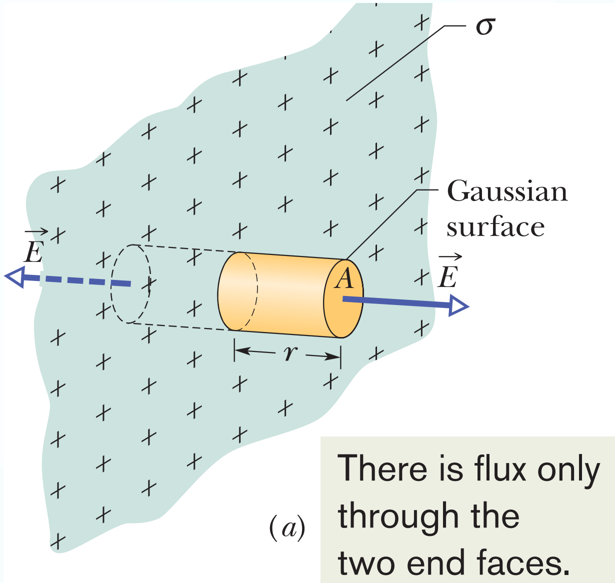


The direction of both electric field opposes the other

$$\rightarrow E = E_{+} - E_{+} = 0$$

Electric field due to a charged <u>sheet</u> (conducting or nonconducting)

A thin infinite, nonconducting (or conducting) sheet with a uniform $+\sigma$ $(\sigma = q_{enc}/A)$

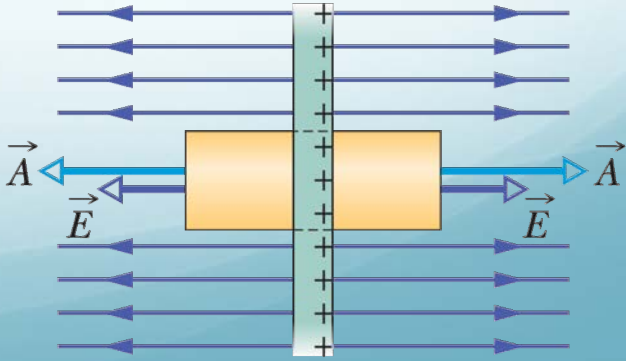


To find ***E*** at distance *r* in front of the sheet:

- Gaussian surface is a closed cylinder with end caps of area *A*, perpendicular the sheet
- →***E*** is perpendicular to the end caps and in outward direction

$$\varepsilon_0 \oint \vec{E} \cdot d\vec{A} = q_{enc}, \rightarrow \varepsilon_0(EA + EA) = \sigma A$$

$$E = \frac{\sigma}{2\varepsilon_0} \qquad \text{(sheet of charge).}$$

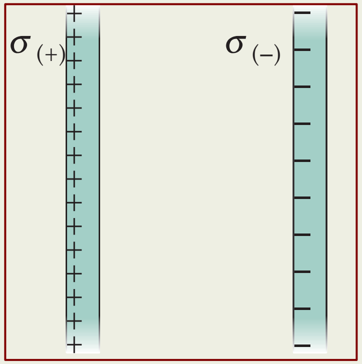


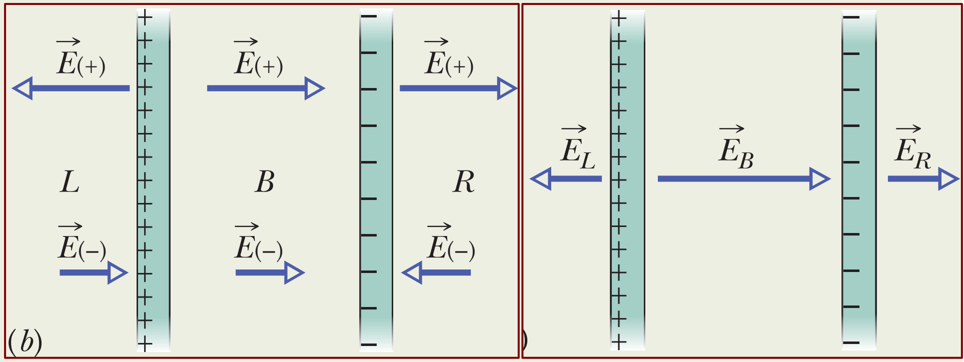
- Because the distance from each flat end of the cylinder to the plane does not appear
  →$E/2\,\varepsilon_0$ at *any* distance from the plane
  →the field is uniform everywhere

Sample Problem

Figure 23-17*a* shows portions of two large, parallel, nonconducting sheets, each with a fixed uniform charge on one side. The magnitudes of the surface charge densities are $\sigma_{(+)} = 6.8\ \mu\text{C/m}^2$ for the positively charged sheet and $\sigma_{(-)} = 4.3\ \mu\text{C/m}^2$ for the negatively charged sheet.

Find the electric field $\vec{E}$ (a) to the left of the sheets, (b) between the sheets, and (c) to the right of the sheets.





Calculations: At any point, the electric field $\vec{E}_{(+)}$ due to the positive sheet is directed *away* from the sheet and, from Eq. 23-13, has the magnitude

$$E_{(+)} = \frac{\sigma_{(+)}}{2\varepsilon_0} = \frac{6.8 \times 10^{-6}\ \text{C/m}^2}{(2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 3.84 \times 10^5\ \text{N/C}.$$

Similarly, at any point, the electric field $\vec{E}_{(-)}$ due to the negative sheet is directed *toward* that sheet and has the magnitude

$$E_{(-)} = \frac{\sigma_{(-)}}{2\varepsilon_0} = \frac{4.3 \times 10^{-6}\ \text{C/m}^2}{(2)(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)} = 2.43 \times 10^5\ \text{N/C}.$$

The resultant fields in these three regions follow from the superposition principle. To the left, the field magnitude is

$$E_L = E_{(+)} - E_{(-)} = 3.84 \times 10^5\ \text{N/C} - 2.43 \times 10^5\ \text{N/C} = 1.4 \times 10^5\ \text{N/C}. \quad \text{(Answer)}$$

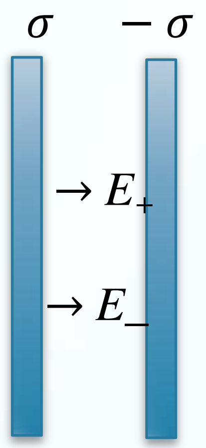
Because $E_{(+)}$ is larger than $E_{(-)}$, the net electric field $\vec{E}_L$ in this region is directed to the left, as Fig. 23-17*c* shows. To the right of the sheets, the electric field has the same magnitude but is directed to the right, as Fig. 23-17*c* shows.

Between the sheets, the two fields add and we have

$$E_B = E_{(+)} + E_{(-)} = 3.84 \times 10^5\ \text{N/C} + 2.43 \times 10^5\ \text{N/C} = 6.3 \times 10^5\ \text{N/C}. \quad \text{(Answer)}$$

Examples:

Q.7 Two parallel <u>conducting</u> <u>sheets</u> carry equal but opposite surface charges of 8.85 nC/m^2. Calculate the electric field between them.



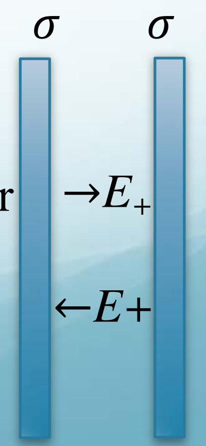
The electric field due to conducting sheet is

$$E = \frac{\sigma}{2\varepsilon_0}$$

The direction of both electric field are the same

$$\rightarrow E = E_+ + E_- = \frac{2\sigma}{2\varepsilon_0} = \frac{8.85\times10^{-9}}{8.85\times10^{-12}} = 1000\mathrm{N/C}$$

Q.8 Two parallel <u>conducting</u> <u>sheets</u> carry equal surface charges of 8.85 nC/m^2. Calculate the electric field between them.



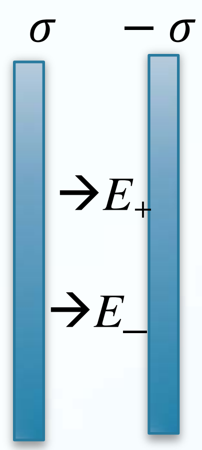
$$E = \frac{\sigma}{2\varepsilon_0}$$

Type equation here.The direction of both electric field opposes the other

$$\rightarrow E = E_+ - E_- = 0$$

Examples:

Q.9 Two parallel nonconducting sheets carry equal but opposite surface charges of 8.85 nC/m². calculate the electric field between them.



The electric field due to nonconducting sheet is

$$E = \frac{\sigma}{2\varepsilon_0}$$

The direction of both electric field are the same

$$\rightarrow E = E_+ + E_- = \frac{2\sigma}{2\varepsilon_0} = \frac{8.85\times10^{-9}}{8.85\times10^{-12}} = 1000\text{N/C}$$

Q.10 An electron is placed near to a nonconducting sheet carrying a surface charge density of 17.7nC/m². Calculate the electronic force acting on the electron.

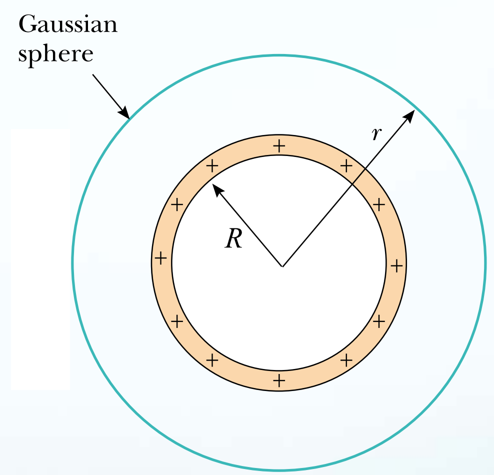
$$E = \frac{\sigma}{2\varepsilon_0} == \frac{17.7\times10^{-9}}{2\times8.85\times10^{-12}} = 1000\text{N/C}$$

$$F = eE = 1.6\times10^{-19}\times1000 = 1.6\times10^{-16}N$$

Electric field due to a charged metallic sphere or shell

Shell theorem

- A shell of uniform charge attracts or repels a charged particle that is outside the shell as if all the shell's charge were concentrated at the center of the shell.
- If a charged particle is located inside a shell of uniform charge, there is no electrostatic force on the particle from the shell.



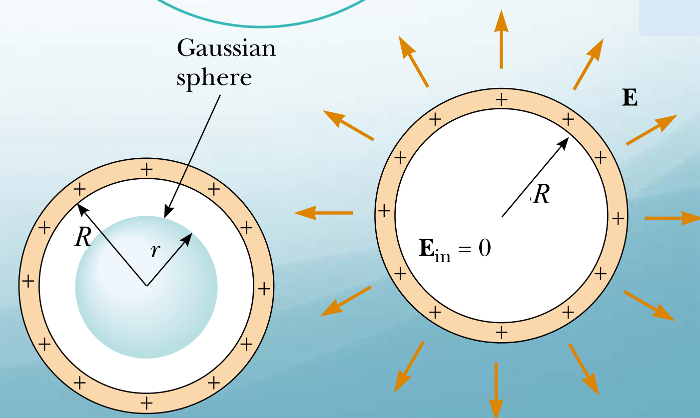
Prove of Shell theorem

- Applying Gauss' law to surface *S*, for which $r \geq R$

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \qquad \text{(spherical shell, field at } r \geq R\text{)}$$

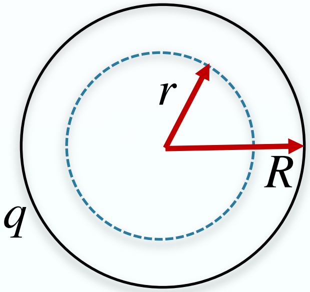
- Applying Gauss' law to surface *S*, for which $r < R$

$$E = 0 \qquad \text{(spherical shell, field at } r < R\text{)},$$



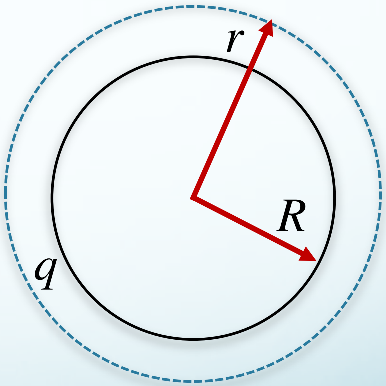
Q.11 A metallic sphere of radius R = 5cm carrying a charge of 5 μC. Calculate the magnitude of the electric field at
(i) r = 3cm
(ii) r = 10cm from the center.

(i) At 3cm from the center:
→inside the sphere
→$q_{enc}=0$
→$E=0$



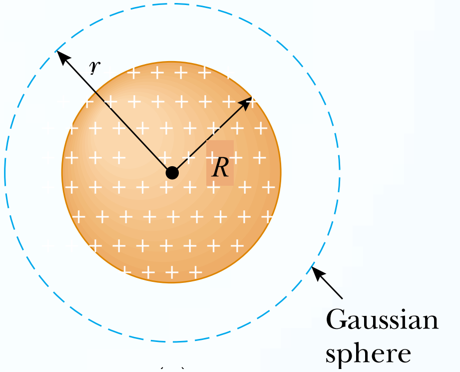
(ii) At 10cm from the center:
→outside the sphere

$$E = \frac{q}{4\pi\varepsilon_o r^2} = \frac{5\times10^{-6}}{4(3.14)(8.85\times10^{-12})(0.10)^2}$$
$$= 4.49 \times 10^6 N/C$$

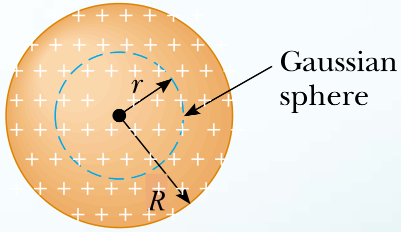


Electric field due to a charged solid (nonconducting) sphere

- If the entire charge lies within a Gaussian surface with $r > R$
  - → E is produces on the Gaussian surface as if the charge were a point charge located at the center
  - → $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2} \quad \text{(spherical shell, field at } r \geq R\text{)}$$



- If a Gaussian surface with $r < R$ & the enclosed charge q'
  - → $$E = \frac{1}{4\pi\varepsilon_0}\frac{q'}{r^2}$$



$$\frac{\begin{pmatrix}\text{charge enclosed by}\\ \text{sphere of radius } r\end{pmatrix}}{\begin{pmatrix}\text{volume enclosed by}\\ \text{sphere of radius } r\end{pmatrix}} = \frac{\text{full charge}}{\text{full volume}}$$

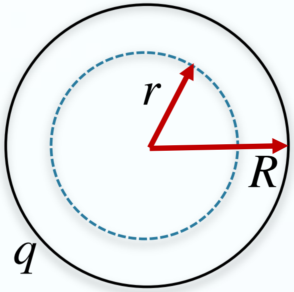
$$\frac{q'}{\frac{4}{3}\pi r^3} = \frac{q}{\frac{4}{3}\pi R^3} \quad \rightarrow \quad q' = q\frac{r^3}{R^3}$$

  - → E inside a uniform sphere of charge is directed radially and has magnitude

$$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right) r$$

Examples:

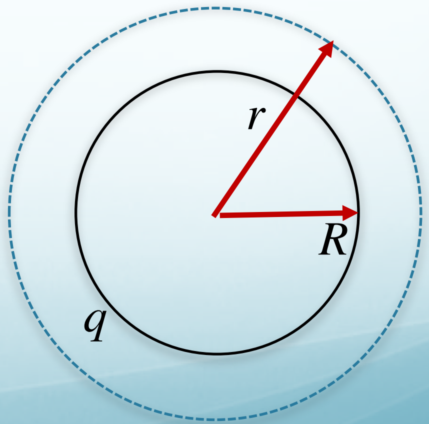
Q.12 A solid sphere of radius 5cm carrying a charge of $q = 5\mu C$. Calculate the magnitude of the electric field at
(i) r = 3cm and
(ii) r = 10cm from the center



(i) At 3cm from the center:
→inside the sphere

$$\rightarrow E = \frac{q}{4\pi\varepsilon_o R^3} r = \frac{5\times10^{-6}}{4(3.14)(8.85\times10^{-12})(0.05)^3} \times 0.03 = 10.8\times10^{6} N/C$$

(i) At 10cm from the center:
→outside the sphere



$$\rightarrow E = \frac{q}{4\pi\varepsilon_o r^2} = \frac{5\times10^{-6}}{4(3.14)(8.85\times10^{-12})(0.10)^2}$$

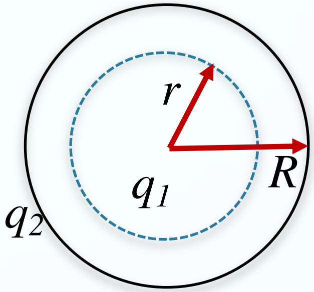
$$= 4.49 \times 10^{6} N/C$$

Examples:

Q.13 A charge of $q_1 = 2\mu C$ is surrounded by a nonconducting sphere of radius 5cm carrying a charge of $q_2 = 5\mu C$. Calculate the magnitude of the electric field at

(i) r = 3cm and

(ii) r = 10cm from the center.



(i) At 3cm from the center:

→inside the sphere

→ $q_{enc} = q_1$

$$\rightarrow E = \frac{q_1}{4\pi\varepsilon_o r^2} = \frac{2\times10^{-6}}{4(3.14)(8.85\times10^{-12})(0.03)^2} = 19.9\times10^6 N/C$$

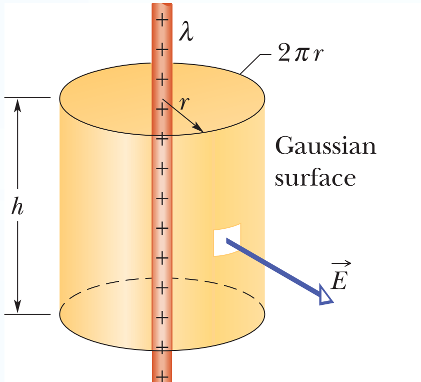
(ii) At 10cm from the center:

→ outside the sphere

→ $q_{enc} = q_1 + q_2$

$$\rightarrow E = \frac{q_1+q_2}{4\pi\varepsilon_o r^2} = \frac{(2+5)\times10^{-6}}{4(3.14)(8.85\times10^{-12})(0.10)^2} = 6.5\times10^6 N/C$$

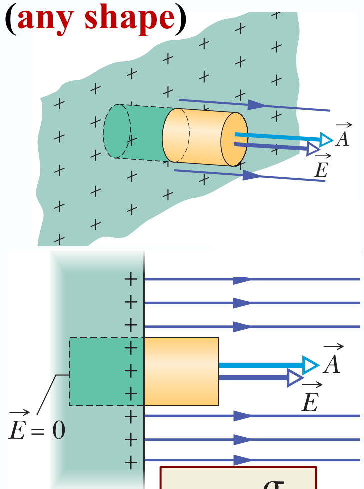
Conducting wire, rod, or line of charge



$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$

$$\lambda = Q/l$$

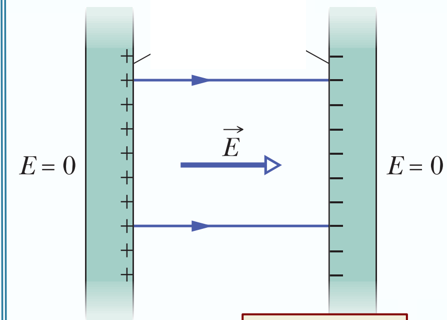
Conducting surface (any shape)



$$E = \frac{\sigma}{\varepsilon_0}$$

$$\sigma = Q/A$$

Conducting plate (with equal &opposite surface area)

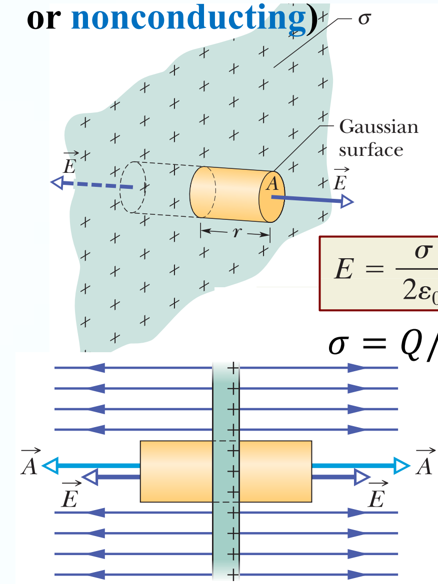


Between $$E = \frac{\sigma}{\varepsilon_0}$$

Outside $$E = 0$$

$$\sigma = Q/A$$

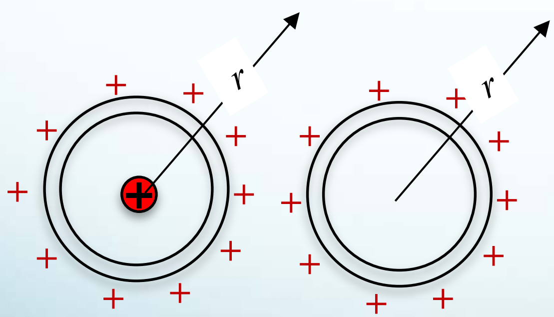
Infinite sheet (conducting or nonconducting)



$$E = \frac{\sigma}{2\varepsilon_0}$$

$$\sigma = Q/A$$

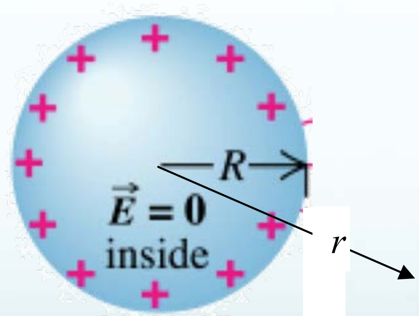
Conducting shell (charged or enclosing charge)



Inside $$E = 0$$

Outside $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

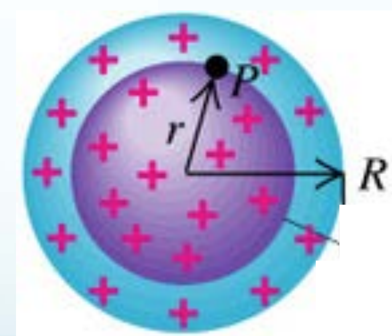
Conducting (metallic) sphere of radius *R*



Inside $r < R$ $$E = 0$$

Outside $r \geq R$ $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

Nonconducting (solid) sphere of radius *R*



Inside $r < R$ $$E = \left(\frac{q}{4\pi\varepsilon_0 R^3}\right)r$$

Outside $r \geq R$ $$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$