# General Physics 2
# SCPH 211

# Chapter 26

# CURRENT AND RESISTANCE

▲ *These power lines transfer energy from the power company to homes and businesses. The energy is transferred at a very high voltage, possibly hundreds of thousands of volts in some cases. Despite the fact that this makes power lines very dangerous, the high voltage results in less loss of power due to resistance in the wires. (Telegraph Colour Library/FPG)*

# Outline:

- Introduction
- Electric Current
- Current Density
- Drift Speed
- Resistance and Resistivity
- Ohm’s Law

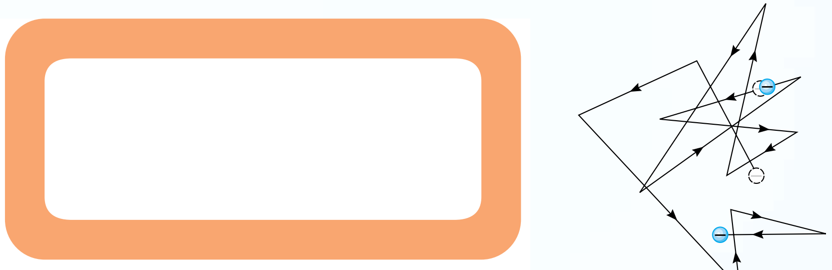
# Introduction

- In the last chapters we discussed the physics of stationary charges (**electrostatics**)
- Now, we will discuss the physics of charges in motion (**electrodynamics** or **electric currents** )
- Examples of electric currents involve many professions:
  - Meteorologists are concerned with lightning and with the less dramatic slow flow of charge through the atmosphere
  - Biologists, physiologists, and engineers working in medical technology are concerned with the nerve currents that control muscles and especially with how those currents can be reestablished after spinal cord injuries
  - Electrical engineers are concerned with countless electrical systems, such as power systems, lightning protection systems, information storage systems, and music systems
  - Space engineers monitor and study the flow of charged particles from our Sun because that flow can wipe out telecommunication systems in orbit and even power transmission systems on the ground
- In this chapter we discuss the basic physics of electric currents and why they can be established in some materials but not in others

# Electric Current

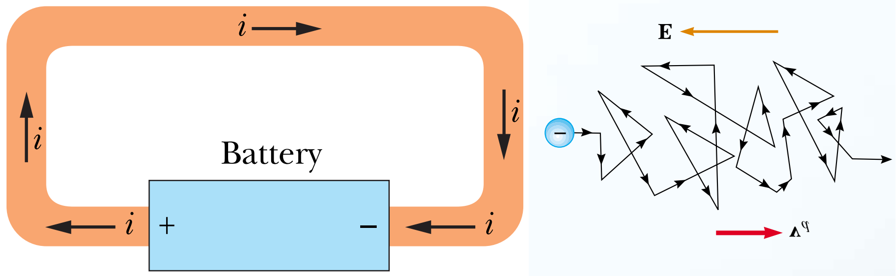
- Although an electric current is a movement of charges, not all moving charges produce electric current

- In Fig.($a$), the free electrons in a copper wire are in random motion
  - The entire loop is at a single potential
    → the electric field is zero at all points inside the copper
  - The loop is in electrostatic equilibrium
  - There is *no net transport* of charge and *no current* through the wire



($a$)

- In Fig. ($b$), a battery was connected between the two ends of the wire
  - An electric field is produced within the loop, from terminal to terminal
  - The field causes charges to move around the loop
  - There is a *net* transport of charge causes current $I$




($b$)

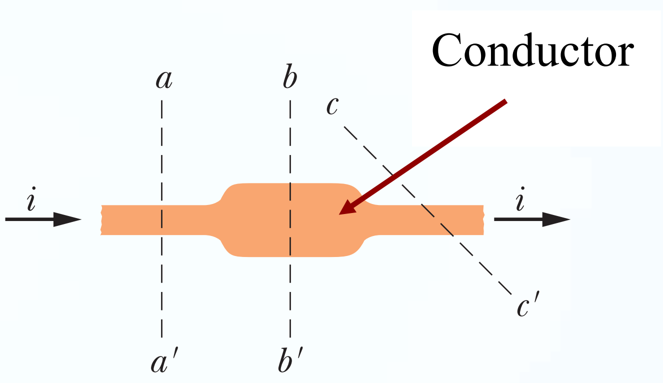
$$i = \frac{dq}{dt} \qquad \text{(definition of current).}$$

- The charge that passes through the plane in a time interval from 0 to $t$ is: $q = \int dq = \int_0^t i\, dt,$

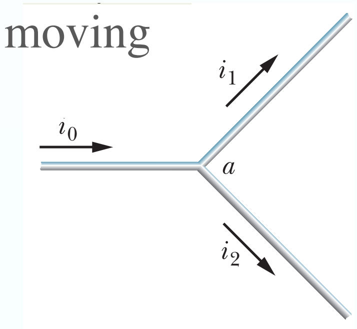
- The SI unit for current is coulomb per second, or ampere (A),

$$1 \text{ ampere} = 1 \text{ A} = 1 \text{ coulomb per second} = 1 \text{ C/s}.$$

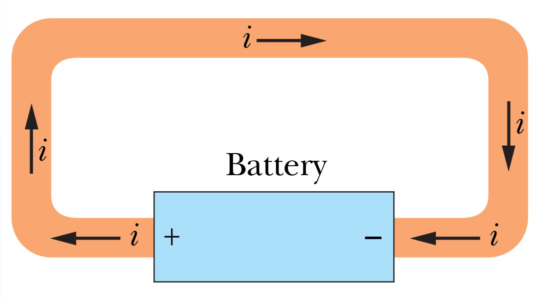
- After a short time, electron flow reaches constant value → current is in its ***steady state***
- Under steady-state conditions, the current is the same for planes *aa'*, *bb'*, and *cc'*




- Current is a scalar because both $q$ & $t$ are scalars
  - We represent a current with an arrow to indicate that charge is moving
  - Such arrows are not vectors (no vector addition)
  - Because charge is conserved, at branch $a$, $i_0 = i_1 + i_2$.




## The Directions of Currents

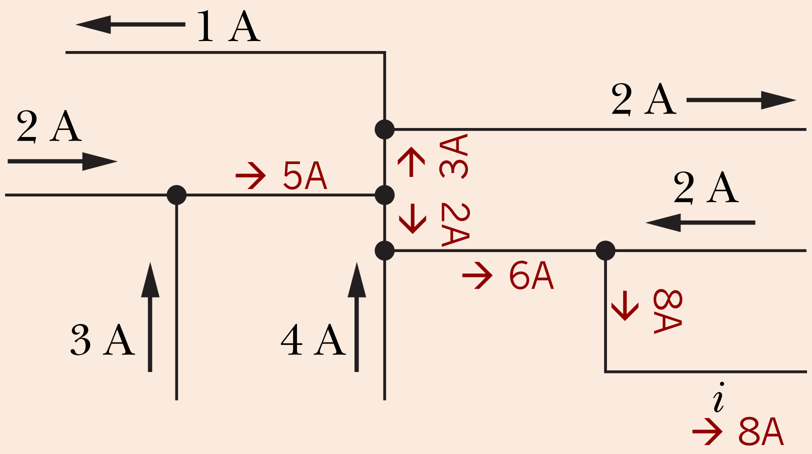



- the current arrows is in the direction in motion of +ve charge
- Positive *charge carriers*, will move away from +ve terminal of battery to the –ve one
- The **E** forces electrons (actual charge carriers) to move in opposite direction of the current arrows, (from–ve terminal to +ve terminal)
- We use the following convention:

> A current arrow is drawn in the direction in which positive charge carriers would move, even if the actual charge carriers are negative and move in the opposite direction.

## CHECKPOINT 1

The figure here shows a portion of a circuit. What are the magnitude and direction of the current $i$ in the lower right-hand wire?




8 A, rightward

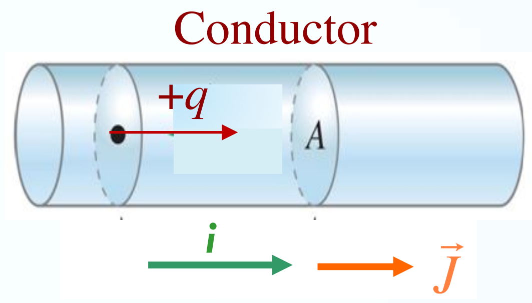
# Current Density

- ***Current density $\vec{J}$:*** the flow of charge through a cross section of the conductor at a particular point
  - $J$ is a <u>vector</u> quantity:

    ***J* Direction**: same direction of the velocity of moving $q$ if $q$ is +ve

    opposite direction the velocity of moving $q$ if $q$ is –ve

    ***J* Magnitude**: $J$ is equal to the current per unit area $J = i/A$




  - If $\overrightarrow{dA}$ is the area vector perpendicular to the element, the total current through the surface is

$$\boxed{i = \int \vec{J} \cdot d\vec{A}.}$$

  - If the current is uniform across the surface & parallel to $dA$, →$J$ is also uniform and parallel to $dA$:

$$i = \int J\,dA = J \int dA = JA,$$

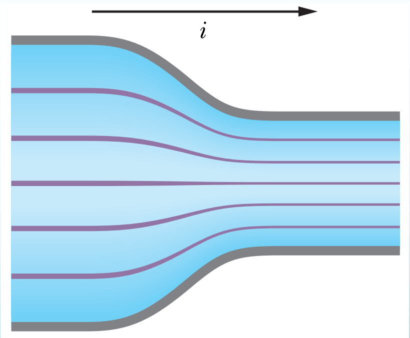
$$J = \frac{i}{A},$$

where $A$ is the total area of the surface

- The SI unit for current density $J$ is the ampere per square meter (A /m$^2$)

- The amount of <u>current</u> <u>cannot</u> change through a conductor

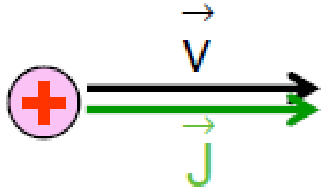
  The <u>current density</u> changes it is greater in the narrower conductor



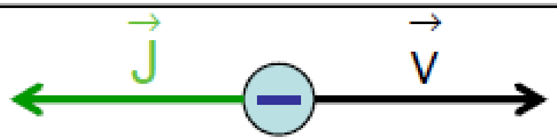

Greater current density is obtained if streamlines are closer

## Current density is a vector quantity

Direction

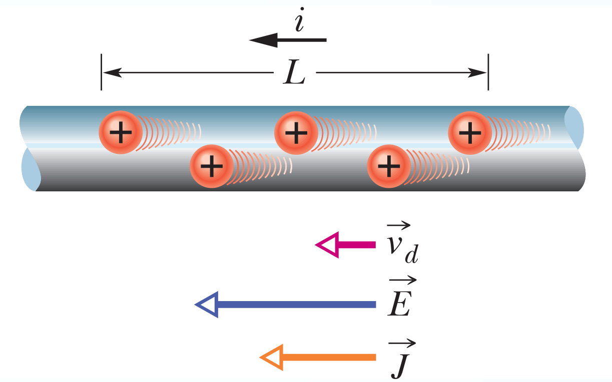



The same direction as the velocity of the moving positive charges.




Opposite to the direction of the velocity of the moving negative charges.

# Drift Speed




- If there is No current flows through the conductor:
  Random motion of conduction electrons → no net motion in any direction

- If there is a flow of current through the conductor:
  Conduction electrons still move randomly, but they *drift* with a ***drift speed*** $v_d$ in opposite direction to $E$

- To relate $v_d$ of electrons with the magnitude $J$:
  - Fig. shows drift of *positive* charge carriers in direction of $E$
  - We assume: charge carriers have $v_d$, $J$ is uniform across wire's cross-sectional area $A$
  - $n$ is the number of carriers per unit volume
  - Number of charge carriers in a length $L$ of wire → $nAL$
  - Total charge of carriers in length $L$, each with charge $e$, → $q = (nAL)e$.
  - Time interval of moving charges in length $L$ → $t = L / v_d$
  - Current $i$ of transfer of charge across a cross section: $i = \frac{q}{t} = \frac{nALe}{L/v_d} = nAev_d$.

$$\rightarrow \quad v_d = \frac{i}{nAe} = \frac{J}{ne}$$

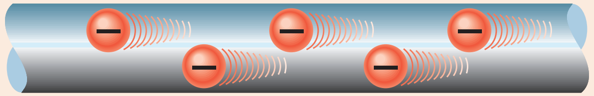
$$\rightarrow \quad \boxed{\vec{J} = (ne)\vec{v}_d.}$$

$$\boxed{\vec{J} = (ne)\vec{v}_d.}$$

- The product *ne*, has SI unit is coulomb per cubic meter ($C/m^3$)
- The product *ne* called the *carrier charge density*
- For +ve carriers, *ne* is +ve → $\vec{J}$ & $\overrightarrow{v_d}$ have the same direction
  For –ve carriers, *ne* is –ve → $\vec{J}$ & $\overrightarrow{v_d}$ have opposite directions

**CHECKPOINT 2**

The figure shows conduction electrons moving leftward in a wire. Are the following leftward or rightward: (a) the current $i$, (b) the current density $\vec{J}$, (c) the electric field $\vec{E}$ in the wire?



Rightward

# Examples:

1. **A cylindrical wire of radius 10 mm has a current of 2 A. Calculate the current density in the wire.**

$$J = \frac{i}{A} = \frac{i}{\pi r^2} = \frac{2}{\pi (0.010)^2} = 6.4 \times 10^3 \, A/m^2$$

# Resistance and Resistivity

- We determine the ***resistance R*** between any two points of <u>conductor</u> by applying potential difference $V$ between those points & measuring current $i$ results

$$\rightarrow \boxed{R = \frac{V}{i} \qquad \text{(definition of } R\text{).}}$$

  - For a given $V$, $R$, $i$
  - The SI unit for $R$ is **ohm** (symbol **Ω**): $1 \text{ ohm} = 1\ \Omega = 1 \text{ volt per ampere} = 1 \text{ V/A.}$

- A ***resistor***: a conductor whose function in a circuit is to provide a specified resistance

- In a circuit diagram, we represent a resistor & resistance by 

- If we focus on $\boldsymbol{E}$ instead of $V$ across a particular resistor, and on $\boldsymbol{J}$ instead of $i$,
  → we deal with the **resistivity $\rho$** of the <u>material</u>:

$$\boxed{\rho = \frac{E}{J} \qquad \text{(definition of } \rho\text{).}} \rightarrow \boxed{\vec{E} = \rho\vec{J}.}$$

  - The SI unit for $\rho$ is ohm.meter (Ω.m) $\dfrac{\text{unit }(E)}{\text{unit }(J)} = \dfrac{\text{V/m}}{\text{A/m}^2} = \dfrac{\text{V}}{\text{A}}\text{m} = \Omega\cdot\text{m.}$

**Table 26-1**

**Resistivities of Some Materials at Room Temperature (20°C)**

| Material | Resistivity, $\rho$ $(\Omega\cdot\text{m})$ | Temperature Coefficient of Resistivity, $\alpha$ $(\text{K}^{-1})$ |
|---|---|---|
| | *Typical Metals* | |
| Silver | $1.62 \times 10^{-8}$ | $4.1 \times 10^{-3}$ |
| Copper | $1.69 \times 10^{-8}$ | $4.3 \times 10^{-3}$ |
| Gold | $2.35 \times 10^{-8}$ | $4.0 \times 10^{-3}$ |
| Aluminum | $2.75 \times 10^{-8}$ | $4.4 \times 10^{-3}$ |
| Manganin[a] | $4.82 \times 10^{-8}$ | $0.002 \times 10^{-3}$ |
| Tungsten | $5.25 \times 10^{-8}$ | $4.5 \times 10^{-3}$ |
| Iron | $9.68 \times 10^{-8}$ | $6.5 \times 10^{-3}$ |
| Platinum | $10.6 \times 10^{-8}$ | $3.9 \times 10^{-3}$ |
| | *Typical Semiconductors* | |
| Silicon, pure | $2.5 \times 10^{3}$ | $-70 \times 10^{-3}$ |
| Silicon, *n*-type[b] | $8.7 \times 10^{-4}$ | |
| Silicon, *p*-type[c] | $2.8 \times 10^{-3}$ | |
| | *Typical Insulators* | |
| Glass | $10^{10} - 10^{14}$ | |
| Fused quartz | $\sim 10^{16}$ | |

- The **conductivity $\sigma$** of a material: is the reciprocal of its resistivity,

$$\boxed{\sigma = \frac{1}{\rho} \qquad \text{(definition of } \sigma\text{).}}$$

  - The SI unit of conductivity is the reciprocal ohm-meter $(\Omega.m)^{-1}$
  - The reciprocal of ohm called mho (℧)
  - Current density could be given by: $\vec{J} = \sigma\vec{E}.$

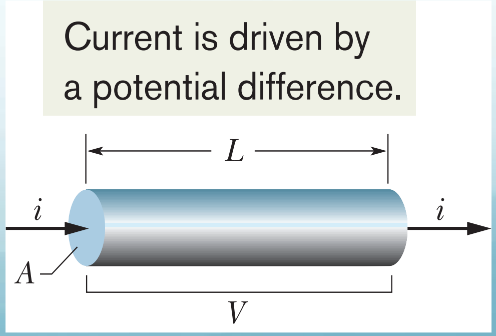
> Resistance is a property of an object. Resistivity is a property of a material.

## Calculating Resistance from Resistivity:

- If we know the resistivity of a substance, we can calculate the resistance of a length of wire made of that substance:
  - let $A$: cross-sectional area of wire, $L$: its length, $V$: potential difference between its ends
  - If $J$ is uniform throughout wire → $E$ constant for all points within the wire

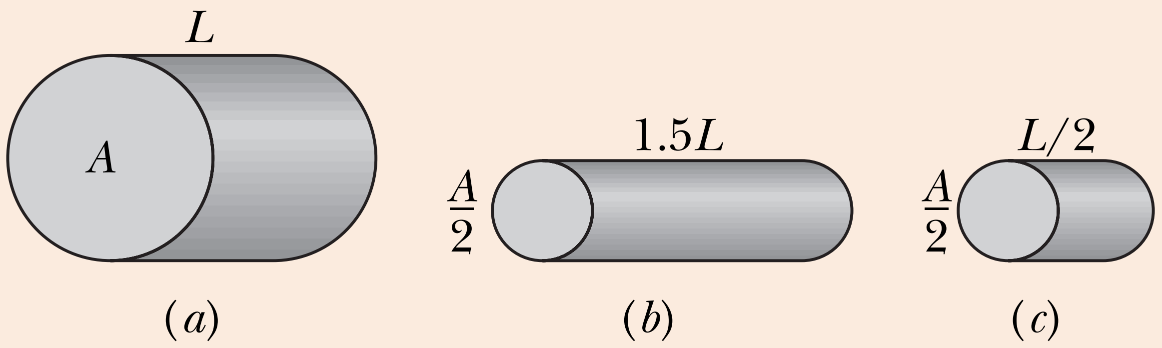
$$\rightarrow E = V/L \quad \text{and} \quad J = i/A \;\rightarrow\; \rho = \frac{E}{J} = \frac{V/L}{i/A}$$

$$\boxed{R = \rho\frac{L}{A}.}$$



## CHECKPOINT 3

The figure here shows three cylindrical copper conductors along with their face areas and lengths. Rank them according to the current through them, greatest first, when the same potential difference $V$ is placed across their lengths.




$$\boxed{i = \frac{V}{R},} \quad \boxed{R = \rho \frac{L}{A}.}$$

$$i = \frac{V}{R} = \frac{VA}{\rho L} = con.\frac{A}{L}$$

$$a)\ \ i = \frac{A}{L}$$

$$b)\ \ i = \frac{A(2)}{2L(3)} = \frac{A}{3L}$$

$$c)\ \ i = \frac{A(2)}{2L} = \frac{A}{L}$$

Answer: a = c, Then b

# Examples:

2. **A 4 Ω resistor is connected to a potential of 12 V. Calculate the current passing through the resistor.**

$$i = \frac{V}{R} = \frac{12}{4} = 3\,A$$

3. **The electric field inside a cylindrical wire of radius 1.2 mm is 0.1 V/m. If the current in the wire is measured to be 16 A, calculate the conductivity of the wire.**

$$\sigma = \frac{J}{E}$$

$$J = \frac{i}{A} = \frac{16}{\pi\left(1.2\times10^{-3}\right)^2} = 3.54\times10^{6}\,A/m^2$$

$$\sigma = \frac{3.54\times10^{6}}{0.1} = 3.54\times10^{7}\,(\Omega.m)^{-1}$$

## Sample Problem

### A material has resistivity, a block of the material has resistance

A rectangular block of iron has dimensions 1.2 cm $\times$ 1.2 cm $\times$ 15 cm. A potential difference is to be applied to the block between parallel sides and in such a way that those sides are equipotential surfaces (as in Fig. 26-8*b*). What is the resistance of the block if the two parallel sides are (1) the square ends (with dimensions 1.2 cm $\times$ 1.2 cm) and (2) two rectangular sides (with dimensions 1.2 cm $\times$ 15 cm)?

**KEY IDEA**

The resistance $R$ of an object depends on how the electric potential is applied to the object. In particular, it depends on the ratio $L/A$, according to Eq. 26-16 ($R = \rho L/A$), where $A$ is the area of the surfaces to which the potential difference is applied and $L$ is the distance between those surfaces.

***Calculations:*** For arrangement 1, we have $L = 15$ cm $= 0.15$ m and

$$A = (1.2 \text{ cm})^2 = 1.44 \times 10^{-4} \text{ m}^2.$$

Substituting into Eq. 26-16 with the resistivity $\rho$ from Table 26-1, we then find that for arrangement 1,

$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega \cdot \text{m})(0.15 \text{ m})}{1.44 \times 10^{-4} \text{ m}^2} 3$$

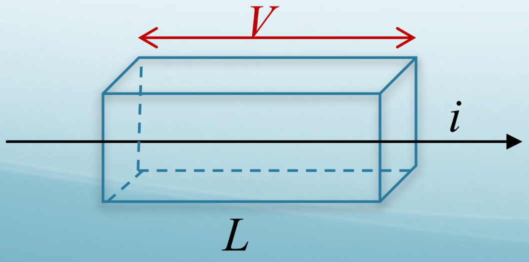
$$= 1.0 \times 10^{-4}\ \Omega = 100\ \mu\Omega. \qquad \text{(Answer)}$$

Similarly, for arrangement 2, with distance $L = 1.2$ cm and area $A = (1.2 \text{ cm})(15 \text{ cm})$, we obtain

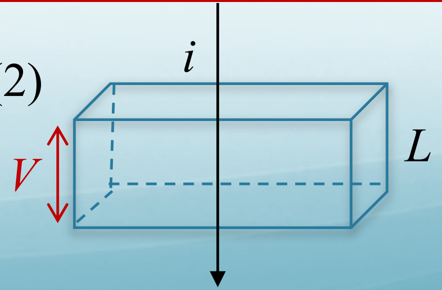
$$R = \frac{\rho L}{A} = \frac{(9.68 \times 10^{-8}\ \Omega \cdot \text{m})(1.2 \times 10^{-2} \text{ m})}{1.80 \times 10^{-3} \text{ m}^2}$$

$$= 6.5 \times 10^{-7}\ \Omega = 0.65\ \mu\Omega. \qquad \text{(Answer)}$$

(1)




(2)



# Examples:

4. **A wire of length 5 cm and cross-sectional area 2 mm$^2$ is connected to a potential of 12 V. If the current passing through the wire is 2 A, determine the resistivity of the wire.**

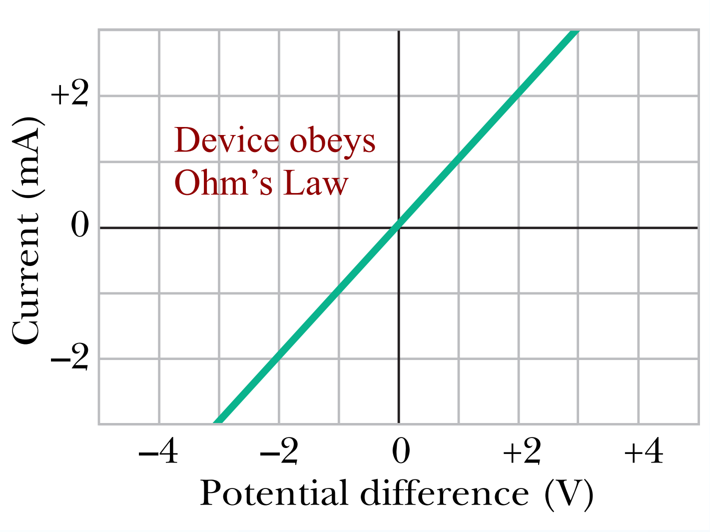
$$\rho = \frac{RA}{L}$$

$$R = \frac{V}{i} = \frac{12}{2} = 6\Omega$$

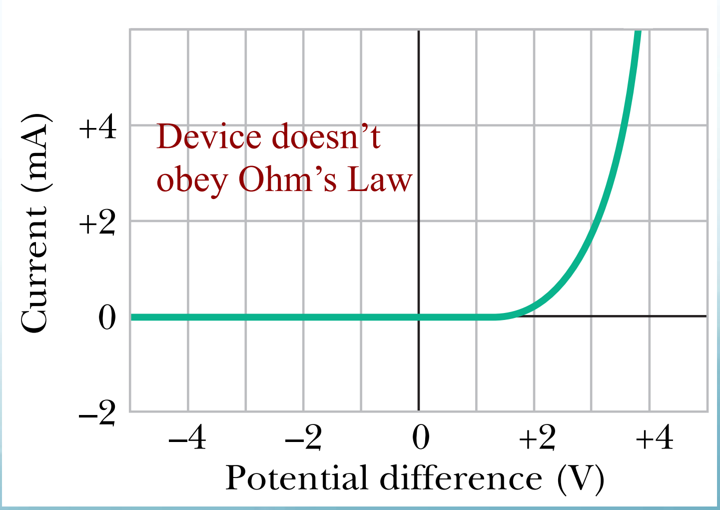
$$\rho = \frac{6 \times 2 \times 10^{-6}}{0.05} = 2.4 \times 10^{-4}\ \Omega.m$$

# Ohm's Law

- A resistor has same resistance regardless magnitude & direction (*polarity*) of applied potential
  → $\boxed{V = iR.}$




- The figure here is a plot of $i$ versus $V$ for one device
  - It is a straight line passing through origin
  - Ratio $i/V$ (slope of straight line) is same for all values of $V$
  - → resistance $R = V/i$ is independent of the magnitude & polarity of $V$




- The figure here is a plot for another device
  - Current exists only if polarity of $V$ is +ve & more than 1.5 V
  - When current does exist, the relation between $i$ & $V$ is not linear
  - The $i$-$V$ relation here depends on the value of $V$

**Ohm's law** is an assertion that the current through a device is *always* directly proportional to the potential difference applied to the device.

A conducting device obeys Ohm's law when the resistance of the device is independent of the magnitude and polarity of the applied potential difference.

$$V = iR.$$

A conducting material obeys Ohm's law when the resistivity of the material is independent of the magnitude and direction of the applied electric field.

$$\vec{E} = \rho\vec{J}.$$

### CHECKPOINT 4

The following table gives the current $i$ (in amperes) through two devices for several values of potential difference $V$ (in volts). From these data, determine which device does not obey Ohm's law.

| Device 1 | | Device 2 | |
|---|---|---|---|
| $V$ | $i$ | $V$ | $i$ |
| 2.00 | 4.50 | 2.00 | 1.50 |
| 3.00 | 6.75 | 3.00 | 2.20 |
| 4.00 | 9.00 | 4.00 | 2.80 |

$$R = \frac{V}{i}$$

| $R$ (Device 1) | $R$ (Device 2) |
|---|---|
| 0.44 | 1.33 |
| 0.44 | 1.36 |
| 0.44 | 1.42 |

Answer: Device 2